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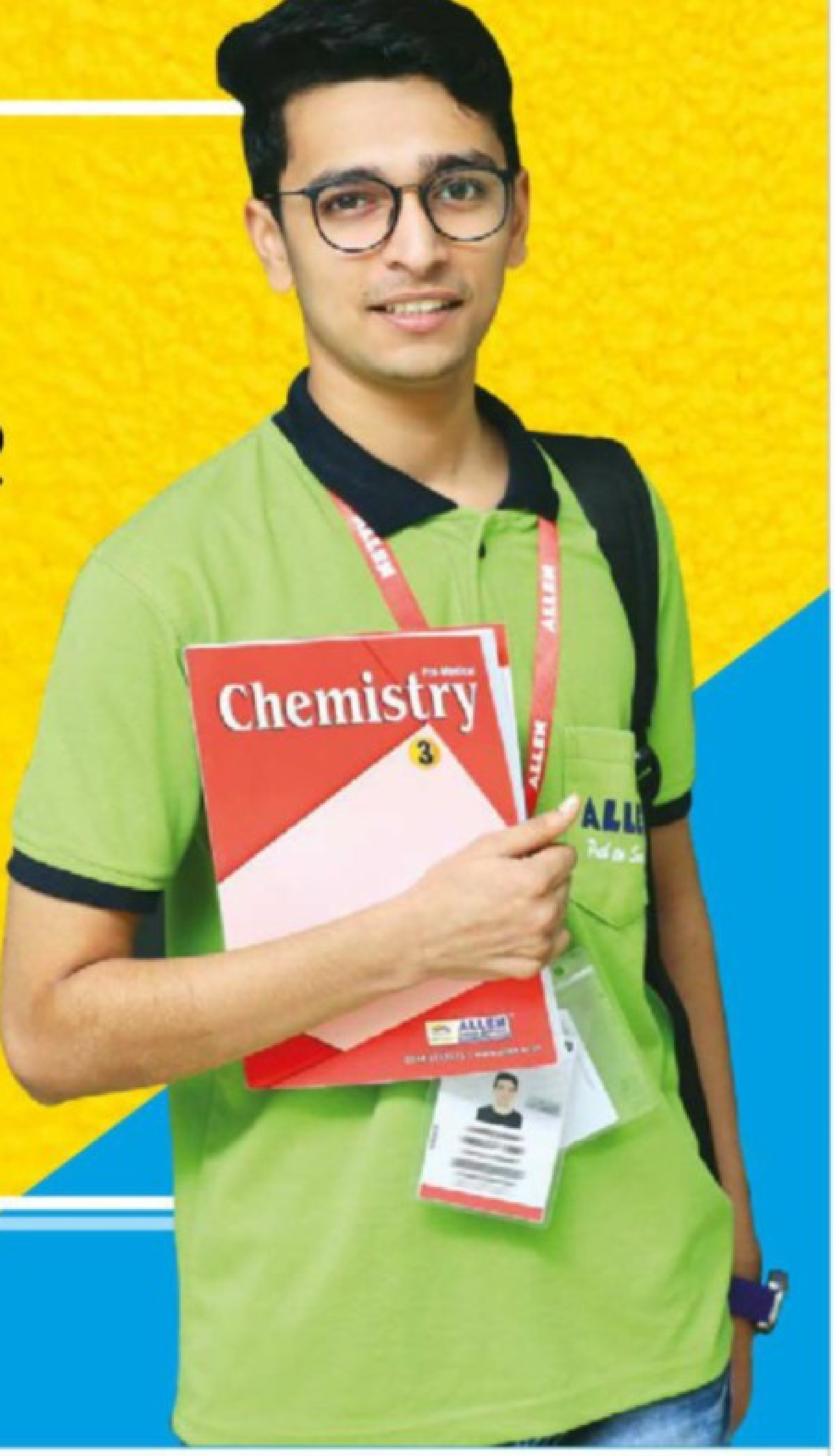

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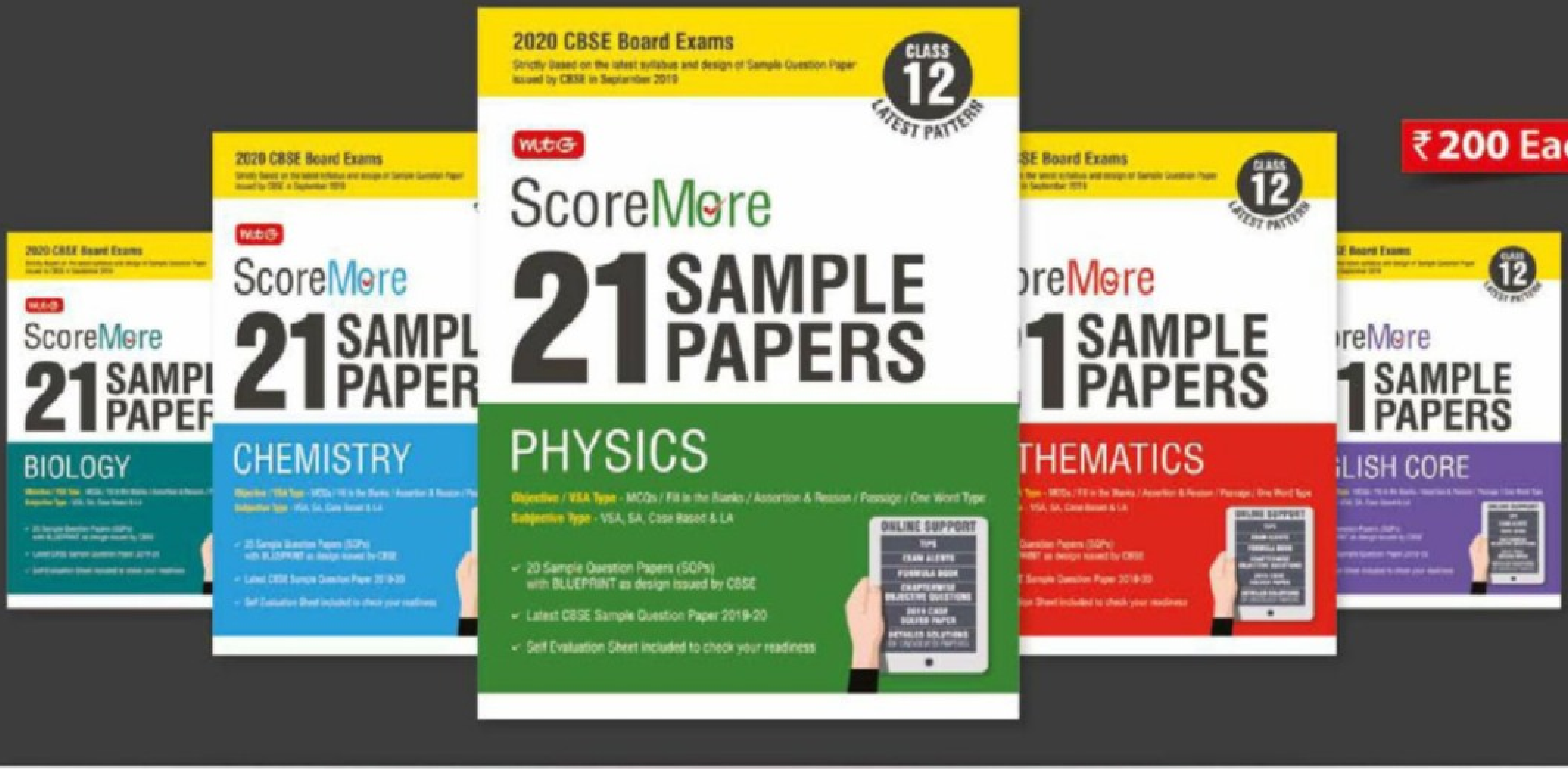

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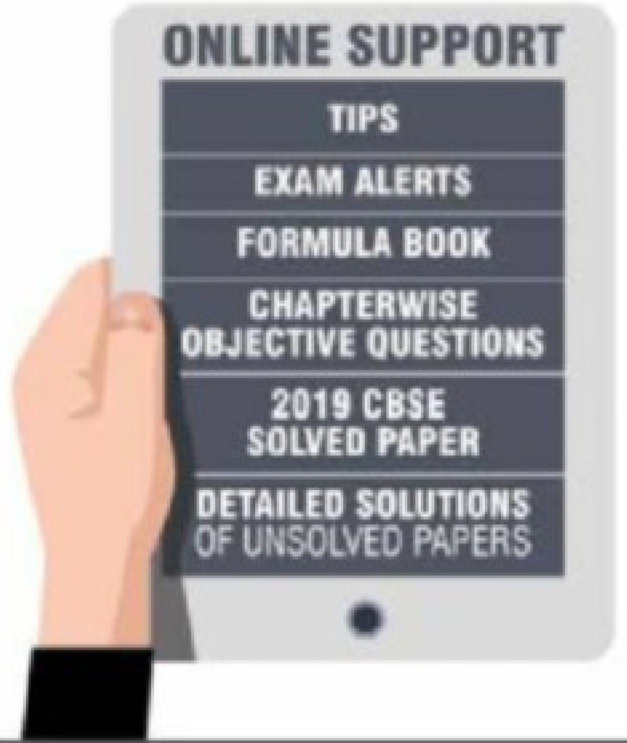

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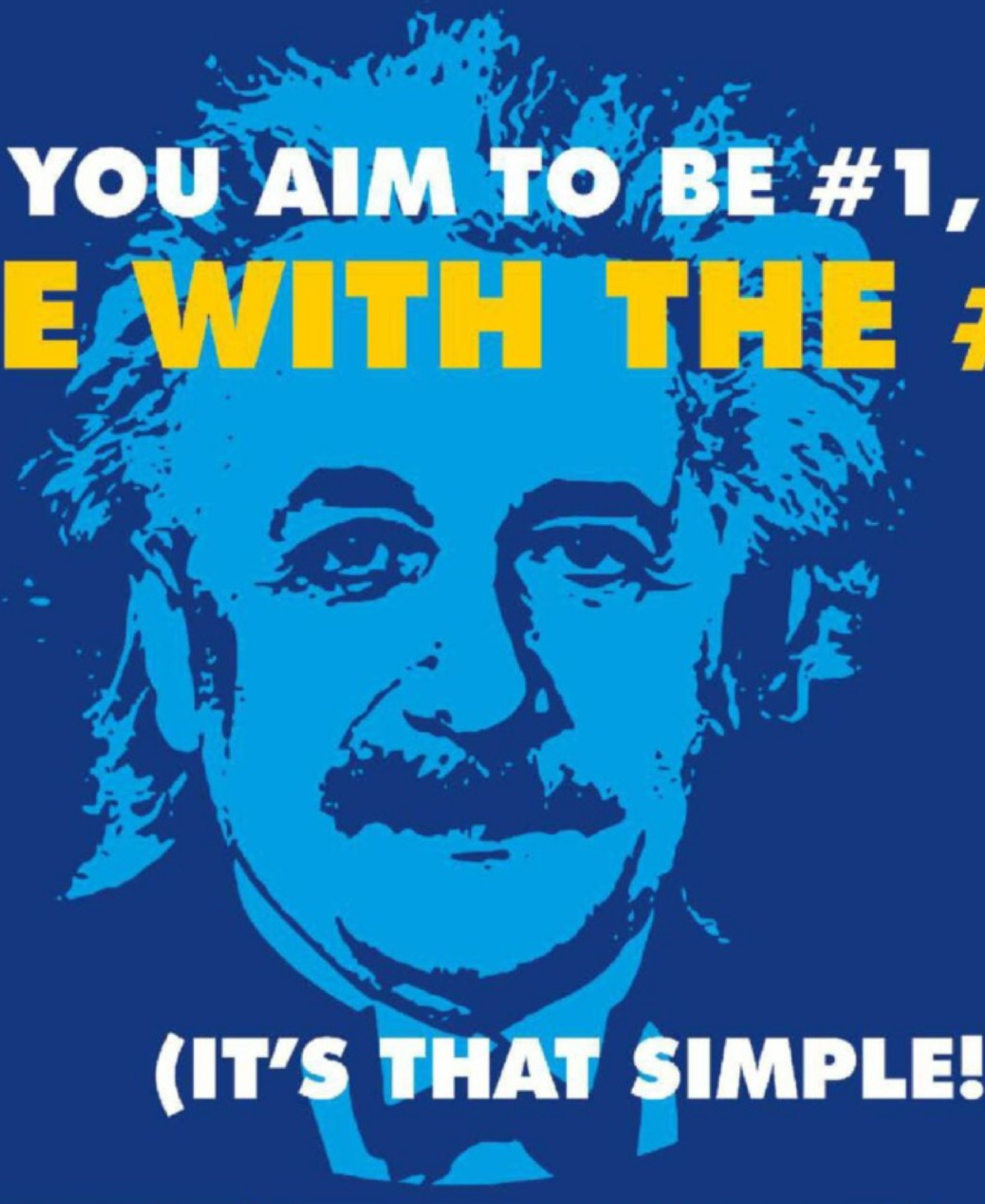

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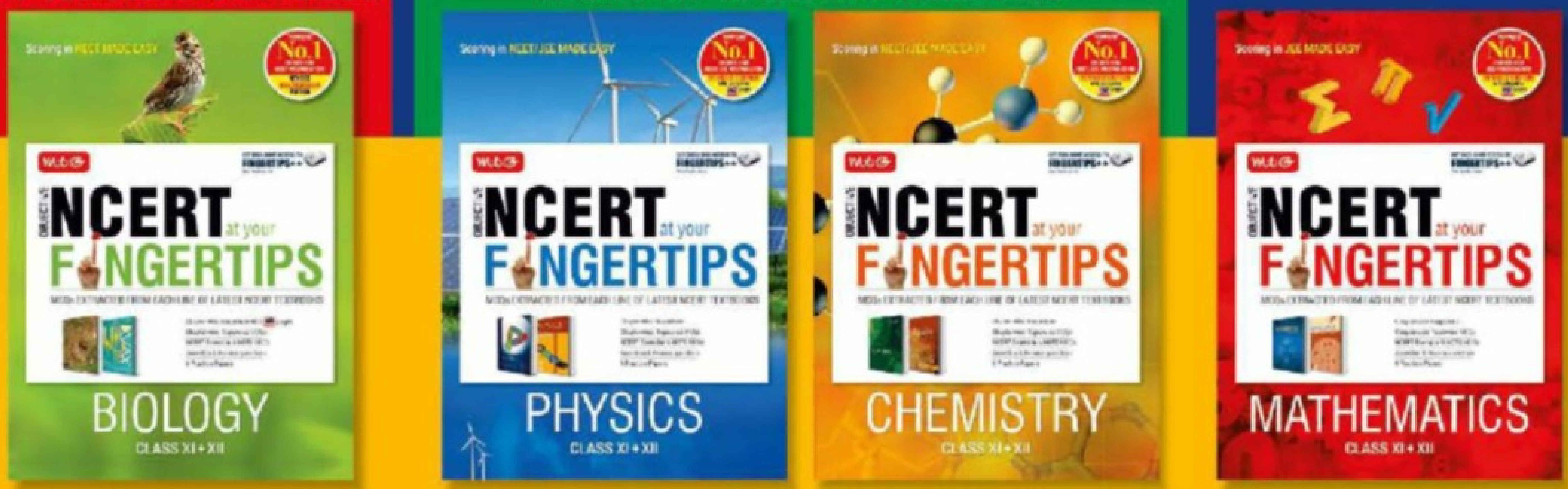
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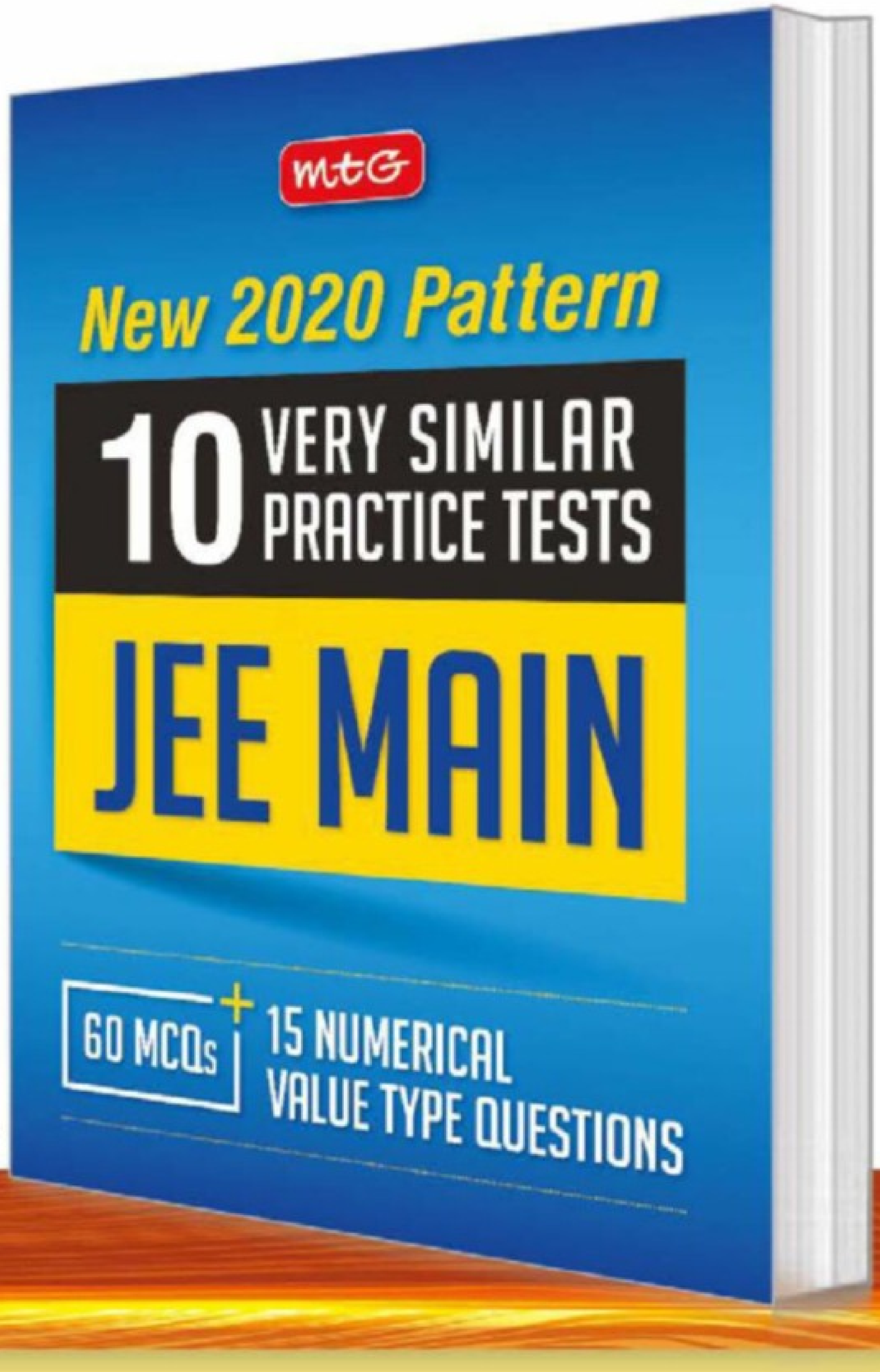
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CONTENTS

Competition Edge

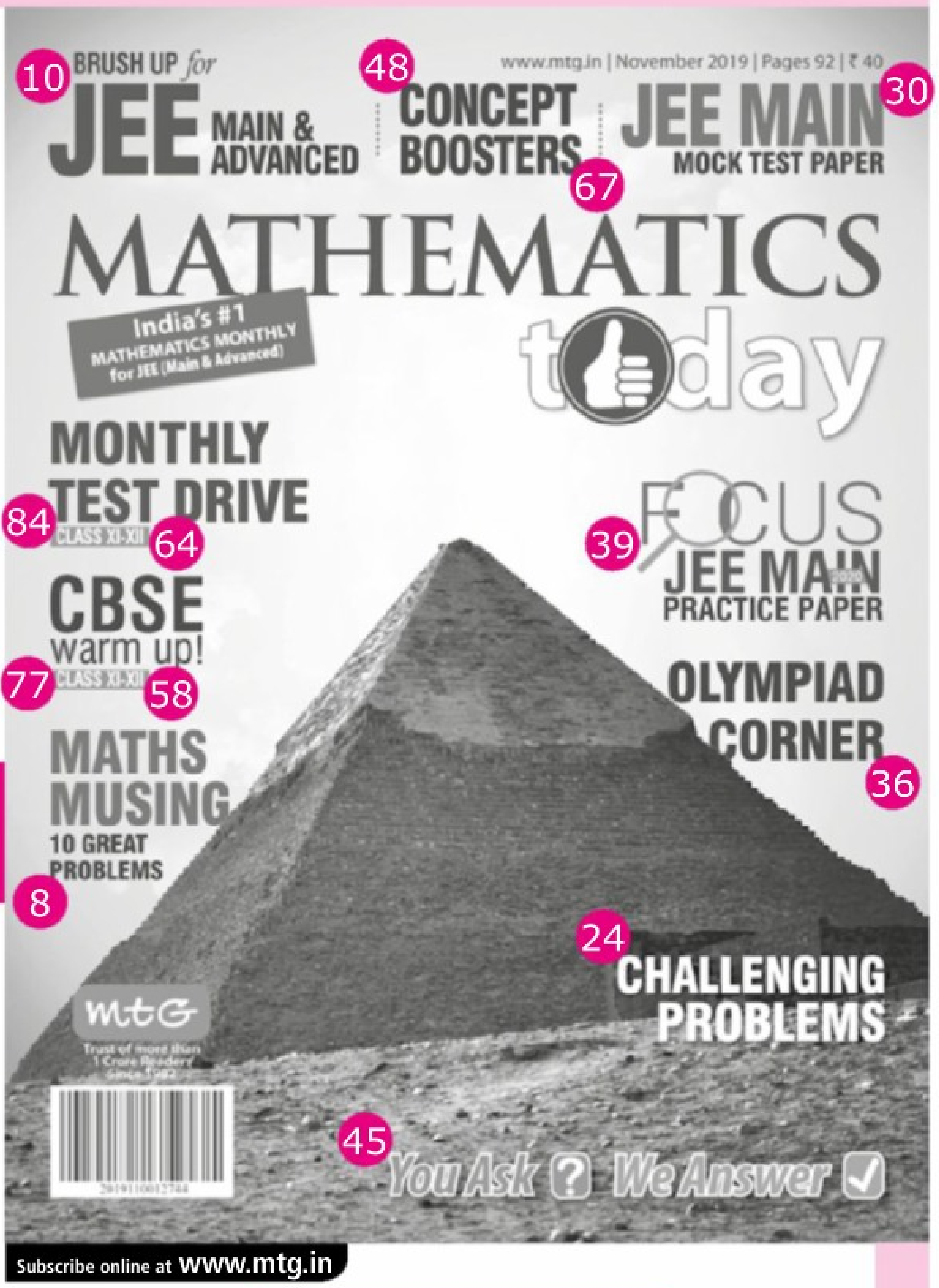
8 Maths Musing Problem Set - 203
10 Brush Up for JEE 2020 (Unit 5)
24 Challenging Problems
30 Mock Test Paper JEE Main 2020 (Series 6)
36 Olympiad Corner
39 Focus JEE Main Practice Paper 2020
44 Maths Musing Solutions
45 You Ask We Answer

Class XI

46 Concept Map
67 Concept Booster
77 CBSE warm-up! (Series 7)
84 Monthly Test Drive (Series 6)

Class XII

47 Concept Map
48 Concept Booster
58 CBSE warm-up! (Series 7)
64 Monthly Test Drive (Series 6)


10 BRUSH UP for JEE MAIN & ADVANCED
48 CONCEPT BOOSTERS 67
www.mtg.in | November 2019 | Pages 92 | ₹ 40
JEE MAIN MOCK TEST PAPER 30
MATHEMATICS today
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MONTHLY TEST DRIVE CLASS XI-XII 84 64
CBSE warm up! CLASS XI-XII 77 58
MATHS MUSING 10 GREAT PROBLEMS 8
39 FOCUS JEE MAIN 2020 PRACTICE PAPER
OLYMPIAD CORNER 36
24 CHALLENGING PROBLEMS
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MATHS MUSING

Maths Musing was started in January 2003 issue of Mathematics Today. The aim of Maths Musing is to augment the chances of bright students seeking admission into IITs with additional study material.

During the last 10 years there have been several changes in JEE pattern. To suit these changes Maths Musing also adopted the new pattern by changing the style of problems. Some of the Maths Musing problems have been adapted in JEE benefitting thousand of our readers. It is heartening that we receive solutions of Maths Musing problems from all over India.

Maths Musing has been receiving tremendous response from candidates preparing for JEE and teachers coaching them. We do hope that students will continue to use Maths Musing to boost up their ranks in JEE Main and Advanced.

PROBLEM
Set 203

JEE MAIN

1. The natural numbers are written as a sequence of digits 1 2 3 4 5 6 7 8 9 10 11 12 . . . . The 2014th digit in the sequence is

(a) 0 (b) 3 (c) 5 (d) 7

2. The sum of the last 4 digits of the natural number 7^{2014} is

(a) 23 (b) 24 (c) 26 (d) 27

3. If a, b, c, d, p are positive numbers such that

$a + b + c + d + p = 8$

$a^2 + b^2 + c^2 + d^2 + p^2 = 16$

then the maximum value of p is

(a) $\frac{4}{5}$ (b) $\frac{8}{11}$ (c) $\frac{16}{5}$ (d) $\frac{28}{11}$

4. If $I=\int_0^2 \max\{\ln(1+x^2), 1\}\,dx$, then

$$I-2[\ln 5+\sqrt{e-1}-\tan^{-1}\sqrt{e-1}+\tan^{-1}2]=$$

(a) 0 (b) 1 (c) 2 (d) –4

5. A man sent 7 letters to his 7 friends. The letters are kept in the addressed envelopes at random. The probability that 3 friends receive correct letters and 4 letters go to wrong destinations is

(a) $\frac{1}{8}$ (b) $\frac{1}{16}$ (c) $\frac{3}{32}$ (d) $\frac{5}{64}$

JEE ADVANCED

6. If $A = (0, 1, -2)$, $B = (2, -1, 0)$, $C = (1, 2, 3)$, then direction ratios of bisector of angle BAC are

(a) –1, 2, 1 (b) 1, –1, 0

(c) 1, 1, 1 (d) 2, –1, 4

COMPREHENSION

Consider the curves $C_1 : y = f(x)$ and $C_2 : y = g(x)$. $f(1) = 1$, $g(2) = 3$, the tangents to C_1 and C_2 at points with equal abscissae intersect on the y-axis, the normals at points with equal abscissae intersect on the x-axis.

7. $f(2) + g(1) =$

(a) 0 (b) 1 (c) 2 (d) 3

8. $\int_1^2 f(x)\, g(x)\, dx =$

(a) $-\frac{1}{2}$ (b) $-\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

NUMERICAL VALUE TYPE

9. Three circles of radii 1, 2, 3 touch each other externally. The radius of the fourth circle which touches all the three circles internally, is

MATRIX MATCH

10. Let N be the number of 4-digit numbers satisfying the condition in column-I and the sum S of the digits of N is in column-II. Match the columns.

	Column-I		Column-II
P.	digits increase from left to right	1.	3
Q.	digits decrease from left to right	2.	9
R.	digits non decreasing from left to right	3.	12
S.	digits non increasing from left to right	4.	18

	P	Q	R	S
(a)	2	1	4	3
(b)	1	2	3	4
(c)	3	4	1	2
(d)	4	3	2	1

See Solution Set of Maths Musing 202 on page no. 44

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(for JEE / Olympiad Aspirants)

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To revise JEE syllabus use a new and difficult book.	For revision always use the same material which is used for studies and use new book to give short duration tests. ***Suggestion***-Give 15 minutes chapter wise test (8 ques. single correct or 5 ques. more than one correct or 2 comprehension or 2 column matching).

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Knowledge Quiz - 24

Find the values of a for which the equation
$(a^2-2)x^3+(a-2)x^2+(a^2-3a+2)x+(a^2-7a+10)=0$,
possess infinite roots

Please send your detailed solution before 15th November 2019 to **quiz@kcseducate.in** along with your name, father's name, class, school, address and contact number.

Winner Knowledge Quiz - 23

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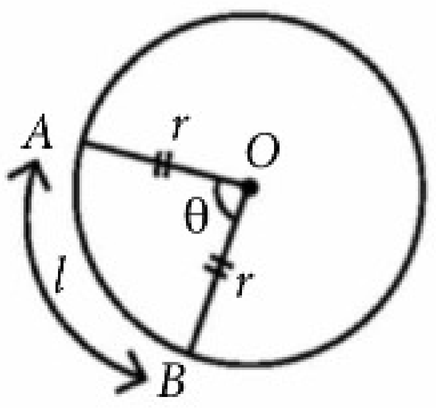
Unit 5

Trigonometry

RADIAN MEASURE

Radian is an angle subtended by a circular arc $\overset{\frown}{AB}$ at the centre of the circle of radius 'r' if $\overset{\frown}{AB} = r$.

Radian Measure and Arc of a Circle :



Let $AB = l$ is the length of an arc of a circle of radius 'r' in given figure and θ is the angle subtended by arc $\overset{\frown}{AB}$ at the centre of the circle, then

$$\theta = \frac{l}{r} \quad \text{or} \quad l = r\theta$$

i.e., length of arc = radius × angle in radian

MEASURE OF ANGLES

There are three systems of measuring angles such as

(1) **Sexagesimal or English system:**

1 right angle = 90 degree (90°)
1° (1 degree) = 60 minutes ($60'$)
$1'$ (1 minute) = 60 seconds ($60''$)

(2) **Centesimal or French system**

1 right angle = 100 grade (100^g)
1 grade (1^g) = 100 minutes ($100'$)
1 minute ($1'$) = 100 seconds ($100''$)

(3) **Circular system**

$$1 \text{ radian} = \frac{180}{\pi} \text{ degree} = 57^\circ\, 17'\, 44.8''$$

$$= 57^\circ\, 17'\, 45'' = 57^\circ \text{ (approximately)}$$

Relationship between Degree (D), Grade (g) and Radian (c)

$180^\circ = 200^g = \pi$ radian

$$\therefore\ 1^\circ = \left(\frac{10}{9}\right)^g = \left(\frac{\pi}{180}\right)^c$$

DOMAIN AND RANGE OF TRIGONOMETRICAL FUNCTIONS

T-Ratio	Domain	Range
$\sin\theta$	$(-\infty, \infty) = R$ (set of real numbers)	$[-1, 1]$
$\cos\theta$	$(-\infty, \infty) = R$ (set of real numbers)	$[-1, 1]$
$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$	Set of real numbers except odd multiples of $\pi/2$	$(-\infty, \infty) = R$ (set of all real numbers)
$\cot\theta = \dfrac{1}{\tan\theta}$	Set of all real numbers except integral multiples of π	$(-\infty, \infty) = R$ (set of all real numbers)
$\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$	Set of all real numbers except integral multiples of π	Set of all real numbers except the values which lies between -1 and 1
$\sec\theta = \dfrac{1}{\cos\theta}$	Set of all real numbers except odd multiples of $\pi/2$	Set of all real numbers except the values which lies between -1 and 1

By : **R. K. Tyagi**, Retd. Principal, HOD Maths, Samarth Shiksha Samiti, New Delhi

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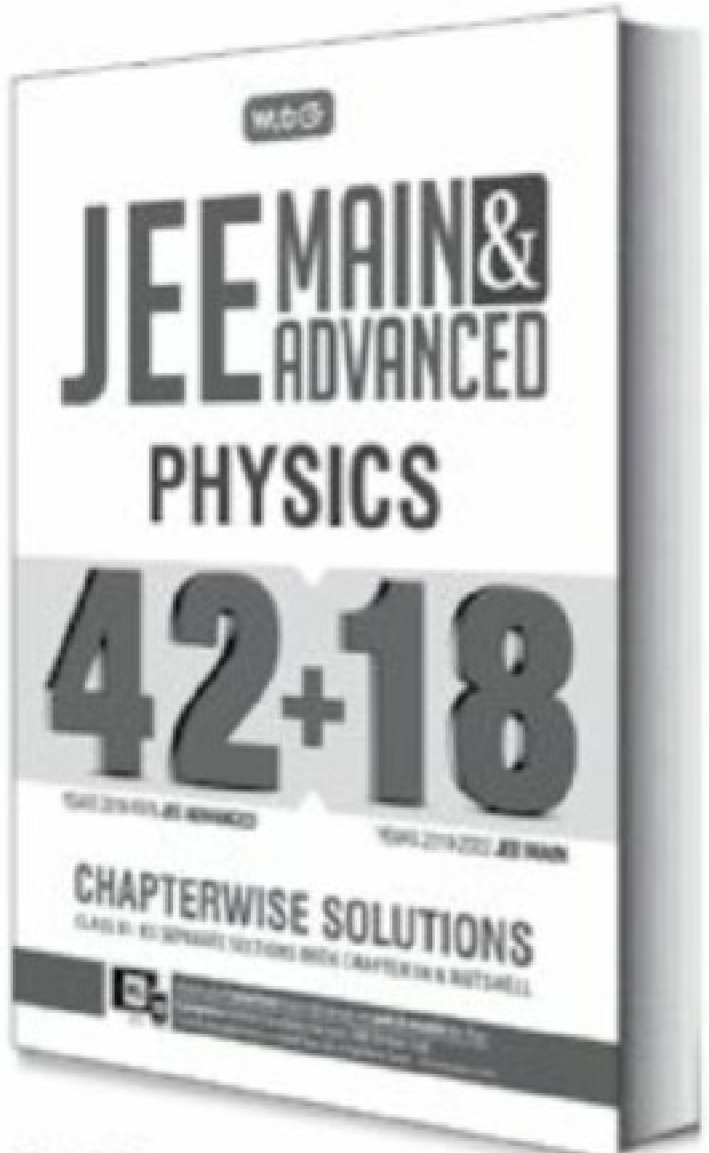
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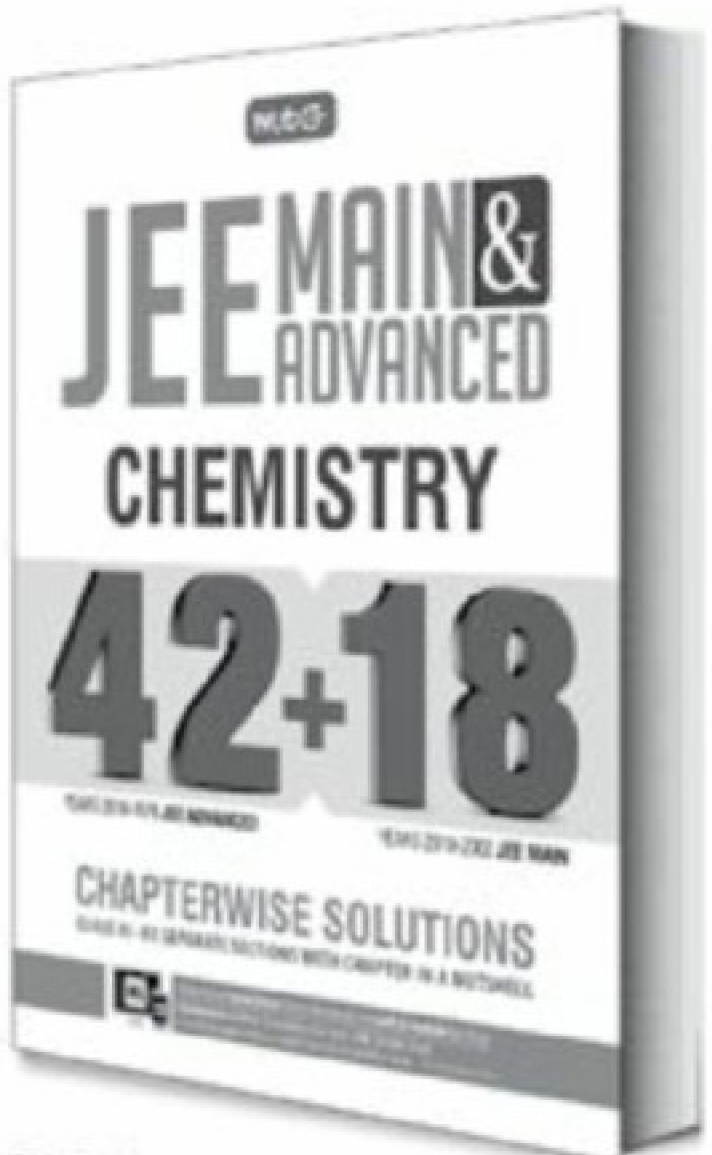
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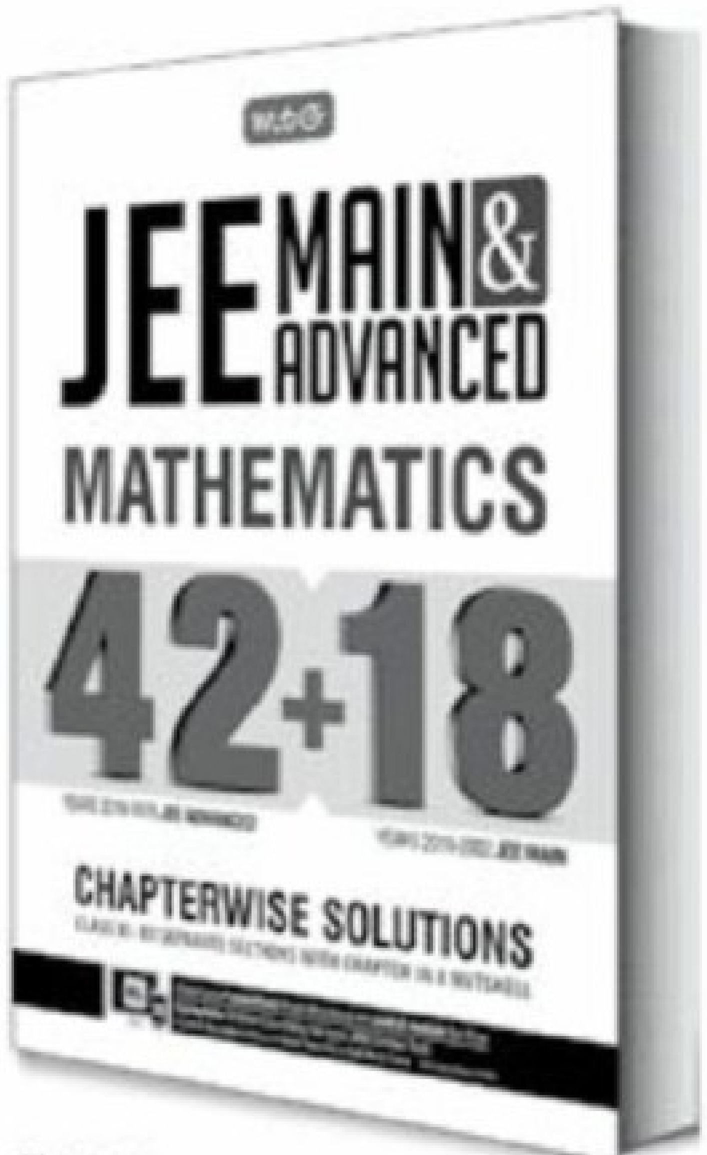
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T-RATIOS OF ALLIED ANGLES

Allied Angles T-Ratio	$\sin\theta$	$\cos\theta$	$\tan\theta$
$-\theta$	$-\sin\theta$	$\cos\theta$	$-\tan\theta$
$\frac{\pi}{2}\mp\theta$	$\cos\theta$	$\pm\sin\theta$	$\pm\cot\theta$
$\pi\mp\theta$	$\pm\sin\theta$	$-\cos\theta$	$\mp\tan\theta$
$\frac{3\pi}{2}\mp\theta$	$-\cos\theta$	$\mp\sin\theta$	$\pm\cot\theta$
$2\pi\mp\theta$	$\mp\sin\theta$	$\cos\theta$	$\mp\tan\theta$

For the whole table, $0 < \theta < \frac{\pi}{2}$

VALUES OF TRIGONOMETRICAL RATIOS OF SOME ANGLES

- $\sin 15^\circ = \sin(45-30)^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$
- $\cos 15^\circ = \cos(45-30)^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$
- $\tan 15^\circ = \tan(45-30)^\circ = 2-\sqrt{3} = \cot 75^\circ$
- $\cot 15^\circ = \frac{1}{\tan 15^\circ} = \frac{1}{2-\sqrt{3}} = 2+\sqrt{3} = \tan 75^\circ$
- $\tan 7\frac{1}{2}^\circ = \cot 82\frac{1}{2}^\circ = \sqrt{2}-\sqrt{3}-\sqrt{4}+\sqrt{6}$
  $= (\sqrt{3}-\sqrt{2})(\sqrt{2}-1)$
- $\cot 7\frac{1}{2}^\circ = \tan 82\frac{1}{2}^\circ = \sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$
  $= (\sqrt{3}+\sqrt{2})(\sqrt{2}+1)$
- $\sin 18^\circ = \cos 72^\circ = \sin 162^\circ = \frac{\sqrt{5}-1}{4}$
- $\cos 18^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4} = \sin 72^\circ$
- $\cos 36^\circ = 2\cos^2 18^\circ - 1 = 1 - 2\sin^2 18^\circ = \frac{\sqrt{5}+1}{4}$
  $= \sin 54^\circ$
- $\sin 36^\circ = \cos 54^\circ = \frac{\sqrt{10-2\sqrt{5}}}{4}$
- $\sin 27^\circ = \cos 63^\circ = \frac{\sqrt{5+\sqrt{5}}-\sqrt{3-\sqrt{5}}}{4}$
- $\cos 27^\circ = \sin 63^\circ = \frac{\sqrt{5+\sqrt{5}}+\sqrt{3-\sqrt{5}}}{4}$

SUM AND DIFFERENCE FORMULA FOR TWO ANGLES

- $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
- $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
- $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
- $\cot(A \pm B) = \frac{\cot A \cot B \mp 1}{\cot B \pm \cot A}$
- $\sin(A+B)\cdot\sin(A-B) = \sin^2 A - \sin^2 B$
  $= \cos^2 B - \cos^2 A$
- $\cos(A+B)\cdot\cos(A-B) = \cos^2 A - \sin^2 B$
  $= \cos^2 B - \sin^2 A$
- $\tan(A+B)\cdot\tan(A-B) = \frac{\tan^2 A - \tan^2 B}{1-\tan^2 A \tan^2 B}$

SUM AND DIFFERENCE FORMULA FOR THREE OR MORE THAN THREE ANGLES

- $\sin(A+B+C) = \cos A \cos B \cos C$
  $(\tan A + \tan B + \tan C - \tan A \tan B \tan C)$
- $\cos(A+B+C) = \cos A \cos B \cos C$
  $(1 - \tan A \tan B - \tan B \tan C - \tan C \tan A)$
- $\tan(A+B+C) = \frac{\sum \tan A - \tan A \tan B \tan C}{1-\sum \tan A \tan B}$
- $\cot(A+B+C) = \frac{\cot A \cot B \cot C - \sum \cot A}{\sum \cot A \cot B - 1}$
- $\sin(A_1 + A_2 + ... + A_n) = \cos A_1 \cos A_2 .... \cos A_n$
  $(S_1 - S_3 + S_5 - S_7 + ...)$
- $\cos(A_1 + A_2 + ... + A_n) = \cos A_1 \cos A_2 ... \cos A_n$
  $(1 - S_2 + S_4 - S_6 + ...)$
- $\tan(A_1 + A_2 + ... + A_n) = \frac{S_1 - S_3 + S_5 - S_7 + ...}{1 - S_2 + S_4 - S_6 + ...}$

where $S_1 = \tan A_1 + \tan A_2 + ... + \tan A_n$

$S_2 = \tan A_1 \tan A_2 + \tan A_2 \tan A_3 + ... + \tan A_{n-1} \tan A_n$

$S_3 = \tan A_1 \tan A_2 \tan A_3 + \tan A_2 \tan A_3 \tan A_4 + ...$

and so on

- $\sin 2A = 2\sin A \cos A = \frac{2\tan A}{1+\tan^2 A}$
- $\cos 2A = \cos^2 A - \sin^2 A = \frac{1-\tan^2 A}{1+\tan^2 A}$
- $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

- $\sin 3A = 3\sin A - 4\sin^3 A$
- $\cos 3A = 4\cos^3 A - 3\cos A$
- $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$
- $\tan 4A = \dfrac{4\tan A - 4\tan^3 A}{1 - 6\tan^2 A + \tan^4 A}$

SUM AND DIFFERENCE IN TERMS OF PRODUCTS

- $2\sin A\cos B = \sin(A + B) + \sin(A - B)$
- $2\cos A\sin B = \sin(A + B) - \sin(A - B)$
- $2\cos A\cos B = \cos(A + B) + \cos(A - B)$
- $2\sin A\sin B = \cos(A - B) - \cos(A + B)$
- $\sin C + \sin D = 2\sin\left(\dfrac{C+D}{2}\right)\cos\left(\dfrac{C-D}{2}\right)$
- $\sin C - \sin D = 2\cos\left(\dfrac{C+D}{2}\right)\sin\left(\dfrac{C-D}{2}\right)$
- $\cos C + \cos D = 2\cos\left(\dfrac{C+D}{2}\right)\cos\left(\dfrac{C-D}{2}\right)$
- $\cos C - \cos D = 2\sin\left(\dfrac{C+D}{2}\right)\sin\left(\dfrac{D-C}{2}\right)$

MAXIMUM AND MINIMUM VALUES OF TRIGONOMETRICAL FUNCTIONS

Let $a = r\cos\alpha$ ... (i) and $b = r\sin\alpha$ ... (ii)

On squaring (i), (ii) and then adding, we get

$r^2 = a^2 + b^2$, *i.e.*, $r = \sqrt{a^2 + b^2}$

$\therefore$ $a\sin\theta + b\cos\theta = r(\sin\theta\cos\alpha + \cos\theta\sin\alpha)$

$= r\sin(\theta + \alpha)$

Again, we know that $-1 \le \sin t \le 1$

$\therefore$ $-1 \le \sin(\theta + \alpha) \le 1$

$\Rightarrow$ $-r \le r\sin(\theta + \alpha) \le r$

$\Rightarrow$ $-r \le a\sin\theta + b\cos\theta \le r$

or $-\sqrt{a^2 + b^2} \le a\sin\theta + b\cos\theta \le \sqrt{a^2 + b^2}$

$\therefore$ Maximum value $= \sqrt{a^2 + b^2}$,

Minimum value $= -\sqrt{a^2 + b^2}$

In a Triangle *ABC*, Maximum and Minimum Value of Trignometrical Functions

- $\sin A + \sin B + \sin C \le \dfrac{3\sqrt{3}}{2}$
- $\sin A\sin B\sin C \le \dfrac{3\sqrt{3}}{8}$
- $\sin\dfrac{A}{2}\sin\dfrac{B}{2}\sin\dfrac{C}{2} \le \dfrac{1}{8}$
- $\sin^2 A + \sin^2 B + \sin^2 C \le \dfrac{9}{4}$
- $\cos A + \cos B + \cos C \le \dfrac{3}{2}$
- $\cos A\cos B\cos C \le \dfrac{1}{8}$
- $\cos\dfrac{A}{2}\cos\dfrac{B}{2}\cos\dfrac{C}{2} \le \dfrac{3\sqrt{3}}{8}$
- $\tan A + \tan B + \tan C \ge 3\sqrt{3}$
- $\tan A\tan B\tan C \ge 3\sqrt{3}$
- $\cot A\cot B\cot C \le \dfrac{1}{3\sqrt{3}}$
- $\tan^2\dfrac{A}{2} + \tan^2\dfrac{B}{2} + \tan^2\dfrac{C}{2} \ge 1$

Conditional Identities

If $A + B + C = \pi$, then

- $\sin 2A + \sin 2B + \sin 2C = 4\sin A\sin B\sin C$
- $\sin A + \sin B + \sin C = 4\cos\dfrac{A}{2}\cos\dfrac{B}{2}\cos\dfrac{C}{2}$
- $\sin A + \sin B - \sin C = 4\sin\dfrac{A}{2}\sin\dfrac{B}{2}\cos\dfrac{C}{2}$
- $\cos 2A + \cos 2B + \cos 2C = -1 - 4\cos A\cos B\cos C$
- $\cos 2A + \cos 2B - \cos 2C = -1 - 4\sin A\sin B\cos C$
- $\cos A + \cos B + \cos C = 1 + 4\sin\dfrac{A}{2}\sin\dfrac{B}{2}\sin\dfrac{C}{2}$
- $\cos A + \cos B - \cos C = -1 + 4\cos\dfrac{A}{2}\cos\dfrac{B}{2}\sin\dfrac{C}{2}$
- $\tan A + \tan B + \tan C = \tan A\tan B\tan C$
- $\cot A\cot B + \cot B\cot C + \cot C\cot A = 1$
- $\tan\dfrac{A}{2}\tan\dfrac{B}{2} + \tan\dfrac{B}{2}\tan\dfrac{C}{2} + \tan\dfrac{C}{2}\tan\dfrac{A}{2} = 1$

PERIODIC FUNCTIONS

A function $f(x)$ is said to be periodic if it assumes the same value after some interval. Thus a function $f(x)$ is called periodic function if $f(x + \alpha) = f(x)$, where least positive value of α is known as period of $f(x)$.

- If $f(x)$ is periodic with period α, then period of $Kf[c(x + \alpha)]$ is $\dfrac{\alpha}{|c|}$.
- Let $f_1(x), f_2(x), \ldots, f_n(x)$ are periodic functions with periods $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n$ respectively then period of $h(x) = a_1 f_1(x) \pm a_2 f_2(x) \pm \ldots \pm a_n f_n(x)$ is LCM of $(\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n)$ provided $h(x)$ is not an even function.

SUM OF TRIGONOMETRICAL SERIES

- $\sin\alpha+\sin(\alpha+\beta)+\sin(\alpha+2\beta)+...+\sin(\alpha+(n-1)\beta)$
$$=\frac{\sin\left(\alpha+\frac{(n-1)\beta}{2}\right)\sin\frac{n\beta}{2}}{\sin(\beta/2)}$$
- $\cos\alpha+\cos(\alpha+\beta)+\cos(\alpha+2\beta)+...+\cos(\alpha+(n-1)\beta$
$$=\frac{\cos\left(\alpha+\frac{(n-1)\beta}{2}\right)\sin\frac{n\beta}{2}}{\sin(\beta/2)}$$
- $\tan\alpha\tan2\alpha+\tan2\alpha\tan3\alpha+....+\tan n\alpha\tan(n+1)\alpha$
$=\cot\alpha\tan(n+1)\alpha-(n+1)$

RESULT IN PRODUCT FORM

- $(\cos\alpha+\cos\beta)(\cos2\alpha+\cos2\beta)(\cos2^2\alpha+\cos2^2\beta)$
$$....(\cos2^n\alpha+\cos2^n\beta)=\frac{\cos2^{n+1}\alpha-\cos2^{n+1}\beta}{2^{n+1}(\cos\alpha-\cos\beta)},n\in N$$
- $(2\cos\theta-1)(2\cos2\theta-1)(2\cos2^2\theta-1)....$
$$(2\cos2^n\theta-1)=\frac{2\cos2^{n+1}\theta+1}{2\cos\theta+1},\ n\in N$$
- $\cos\theta\cos2\theta\cos2^2\theta\cos2^3\theta.....\cos2^{n-1}\theta$
$$=\begin{cases}\dfrac{\sin2^n\theta}{2^n\sin\theta}, & \text{if }\theta\ne n\pi,n\in I\\ \dfrac{1}{2^n}, & \text{if }\theta=\dfrac{\pi}{2^n+1}\\ -\dfrac{1}{2^n}, & \text{if }\theta=\dfrac{\pi}{2^n-1}\end{cases}$$

TRIGONOMETRIC EQUATIONS

An equation involving one or more t-ratios of unknown angle is called trigonometric equation. It is well known fact that trigonometrical identity is satisfied for every value of unknown angle whereas, trigonometric equation is satisfied only for some value of unknown angles. For example $\sec^2x-\tan^2x=1$ is a trigonometric identity as it is satisfied for each value of $x\in R$.

Solution of Trigonometric Equation

A value of unknown angle which satisfies the given equation is called its solution.

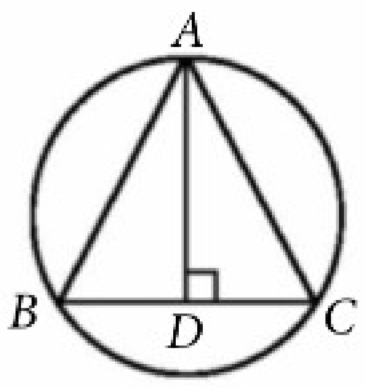
General Solution:

Since trigonometric functions are periodic, the solution of trigonometric equations can be generalised by using the periodicity of trigonometric functions. A solution set containing all possible solutions of a trigonometric equation is said to be its general solution. The following table gives some standard trigonometric equations and their solutions.

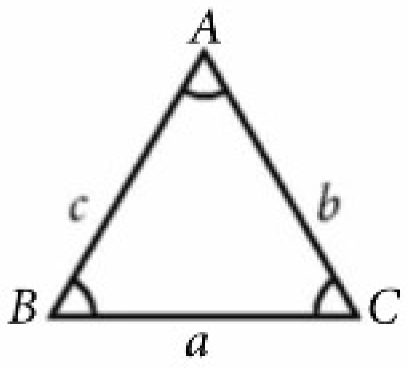
	Equation	General solution where $n\in I$
(1)	$\sin\theta=0,\ \tan\theta=0$	$\theta=n\pi$
(2)	$\cos\theta=0$	$\theta=(2n+1)\frac{\pi}{2}$
(3)	$\sin\theta=\sin\alpha\ \left(-\frac{\pi}{2}\le\alpha\le\frac{\pi}{2}\right)$	$\theta=(-1)^n\alpha+n\pi$
(4)	$\cos\theta=\cos\alpha\ (0\le\alpha\le\pi)$	$\theta=2n\pi\pm\alpha$
(5)	$\tan\theta=\tan\alpha\ \left(-\frac{\pi}{2}<\alpha<\frac{\pi}{2}\right)$	$\theta=n\pi+\alpha$
(6)	$\sin^2\theta=\sin^2\alpha,\ \cos^2\theta=\cos^2\alpha,\ \tan^2\theta=\tan^2\alpha$	$\theta=n\pi\pm\alpha$

PROPERTIES OF TRIANGLES

- **Sine Formula :** In a triangle ABC,
$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=2R$$
where $R=\frac{abc}{4\Delta}$ is radius of the circumcentre of ΔABC whose area is Δ.



- **Cosine Formula :**
$$\cos A=\frac{b^2+c^2-a^2}{2bc}$$
Similarly other results.

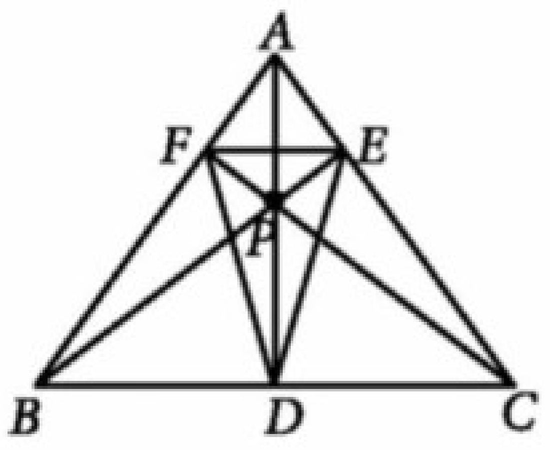


- **Projection Formula :**
$a=c\cos B+b\cos C,\ b=a\cos C+c\cos A,$
$c=a\cos B+b\cos A$
- **Half Angle Formula :**
$$\sin\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{bc}},\ \cos\frac{A}{2}=\sqrt{\frac{s(s-a)}{bc}}$$
$$\tan\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$ and similarly other results.
- **Napier's Analogy (Tangent Rule) :**
$$\tan\frac{A-B}{2}=\frac{a-b}{a+b}\cot\frac{C}{2}$$
Similarly other results.
- **Area of Triangle in Different Forms :**
If a, b, c are sides and A, B, C are angles of triangle opposite to the sides a, b and c respectively, then area of triangle is given by

$$\Delta = \frac{1}{2}ab\sin C = \frac{1}{2}ac\sin B = \frac{1}{2}bc\sin A$$

$$\Delta = \frac{c^2\sin A\sin B}{2\sin C} = \frac{b^2\sin A\sin C}{2\sin B} = \frac{a^2\sin B\sin C}{2\sin A}$$

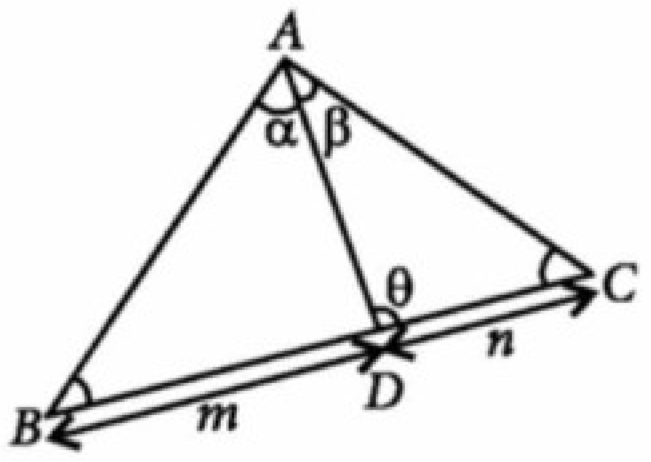
PEDAL TRIANGLE



Let ABC be any triangle, AD, BE, EF are ⊥'s drawn from vertices A, B, C to the opposite sides BC, CA and AB respectively. The point of concurrency of these ⊥'s is called orthocentre of ΔABC. The triangle formed by joining the feet of these ⊥'s is called the pedal triangle. In the adjoining figure, ΔDEF is pedal triangle of ΔABC.

- Sides of pedal ΔDEF are
  $EF = a\cos A = 2R\sin A\cos A = R\sin 2A$
  $FD = b\cos B = 2R\sin B\cos B = R\sin 2B$
  $DE = c\cos C = 2R\sin C\cos C = R\sin 2C$
- Angles of pedal triangle are $\pi - 2A$, $\pi - 2B$, $\pi - 2C$.
- Area of pedal triangle = $2\Delta\cos A\cos B\cos C$

m-n cot Rule

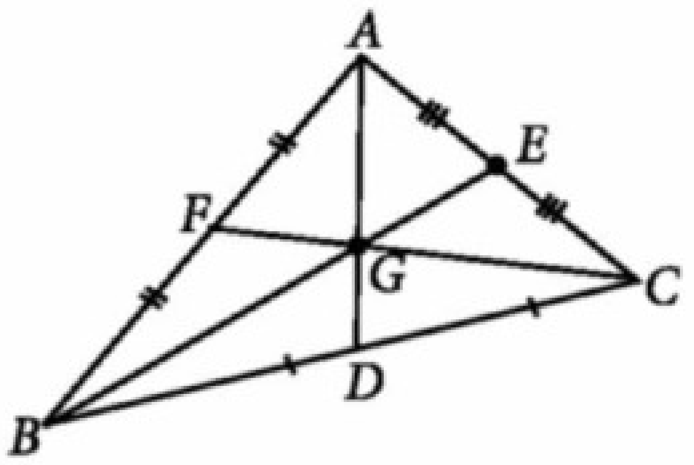


If D is a point on the side BC of ΔABC such that $BD : DC = m : n$

Let $\theta = \angle ADC$, $\angle DAB = \alpha$, $\angle CAD = \beta$ then,

- $(m + n)\cot\theta = m\cot\alpha - n\cot\beta$
- $(m + n)\cot\theta = n\cot B - m\cot C$

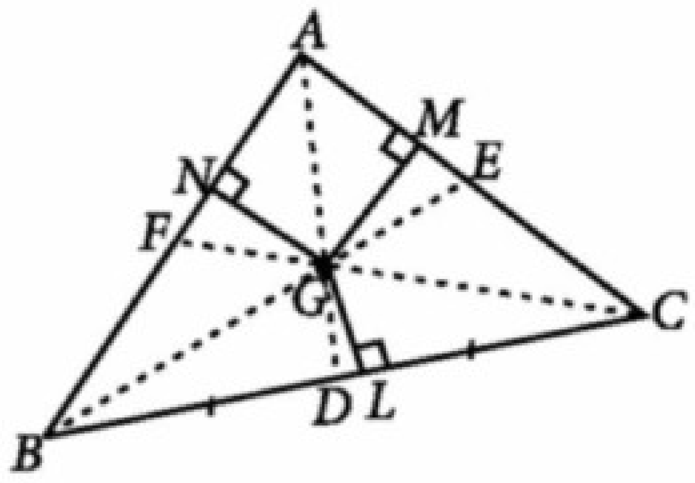
CENTROID AND MEDIAN OF A TRIANGLE



- Three medians of a triangle are concurrent and the point of concurrency of the medians of any triangle is called centroid of the triangle.
- Centroid divides the medians in the ratio 2 : 1.
- The distance of centroid from vertices of triangle ABC is GA, GB and GC where

  $$GA = \frac{1}{3}\sqrt{2b^2 + 2c^2 - a^2},\ GB = \frac{1}{3}\sqrt{2a^2 + 2c^2 - b^2},$$

  $$GC = \frac{1}{3}\sqrt{2a^2 + 2b^2 - c^2}$$

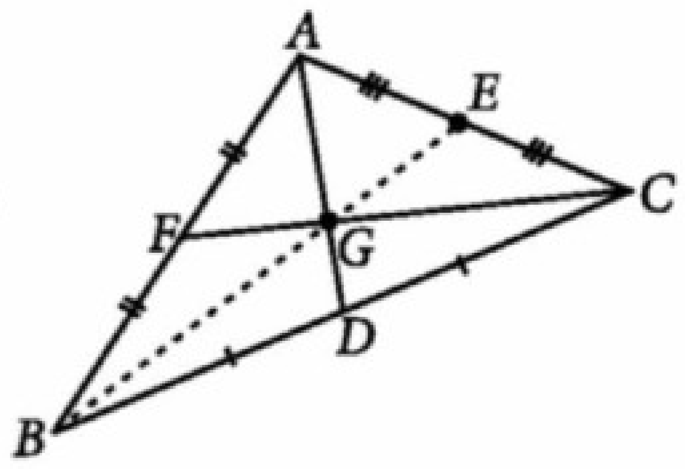


The Distance of Centroid from the Sides : The distance of centroid from BC, AC and AB are respectively GL, GM and GN where

$$GL = \frac{2\text{Area of } \Delta ABC}{3a} = \frac{2\Delta}{3a}$$

$$GM = \frac{2\Delta}{3b},\ GN = \frac{2\Delta}{3c}$$

APPOLONIUS THEOREM

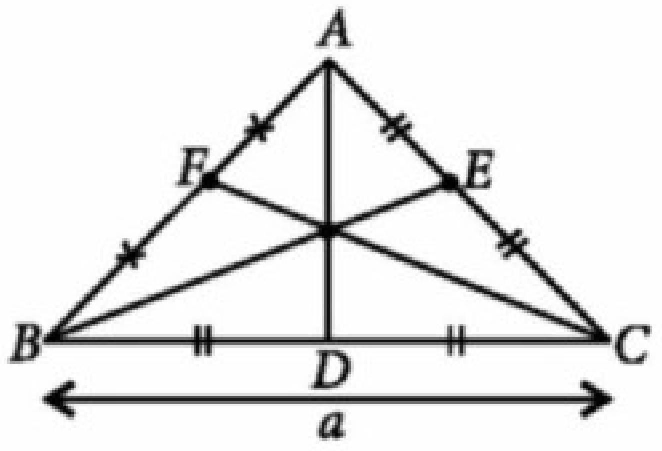


In a triangle if AD is median through A, then

$AB^2 + AC^2 = 2(AD^2 + BD^2)$

or $2(AD^2 + DC^2)$

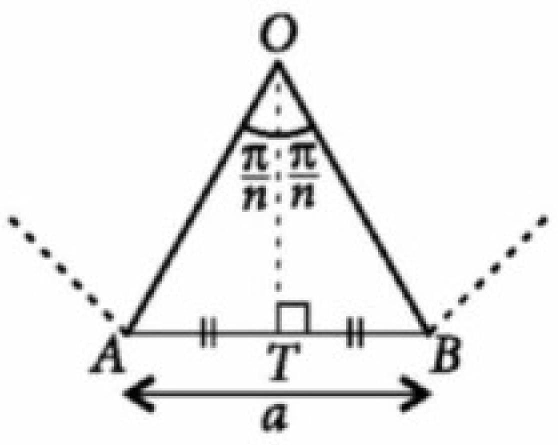
as $BD = DC$ and similarly the other two results.



Note : If AD, BE and CF are medians through A, B, C to the opposite sides BC, AC and AB respectively, then $(AD)^2 + (BE)^2 + (CF)^2$

$$= \frac{3}{4}(a^2 + b^2 + c^2)$$

POLYGONS (INSCRIBED AND CIRCUMSCRIBED)



- Let 'a' be the side of regular polygon of n sides
  $OT = r$ (radius of incircle of polygon)

  $$= \frac{a}{2}\cot\frac{\pi}{n}$$

- $OA = OB = R$(radius of circumcircle of polygon)

  $$= \frac{a}{2}\operatorname{cosec}\frac{\pi}{n}$$

- Area of regular polygon of n sides inscribed in a circle of radius 'r' = number of sides × Area of ΔOAB

  $$= n \times \frac{1}{2}AB \times OT = n\left(\frac{1}{2}2AT\right)OT = nAT\cdot OT$$

  $$= \frac{na^2}{4}\cot\frac{\pi}{n}\text{(in terms of side of } \Delta)$$

  $$= nr^2\tan\frac{\pi}{n}\text{(in terms of in-radius)}$$

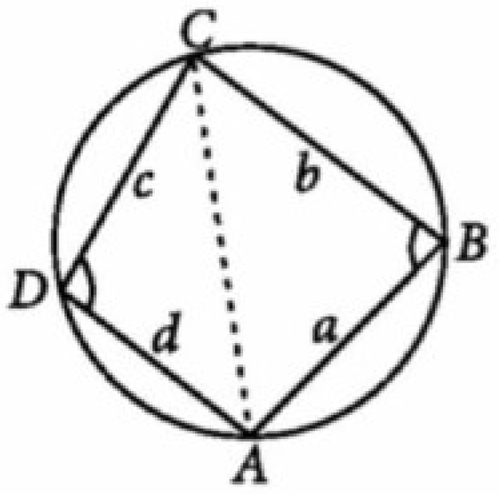
  $$= \frac{nR^2}{2}\sin\frac{2\pi}{n}\text{(in terms of circumradius)}$$

- Side of inscribed polygon $=2R\sin\frac{\pi}{n}$
- Side of circumscribed polygon $=2r\tan\frac{\pi}{n}$
- Sum of radii of inscribed and circumscribed circles of n sided polygon $=r+R=\frac{a}{2}\cot\frac{\pi}{2n}$

CYCLIC QUADRILATERAL

A quadrilateral is said to be cyclic quadrilateral if all its vertices lies on a circle.

- If A, B, C, D are vertices of a cyclic quadrilataral taken in order then



1. $\angle A + \angle C = \angle B + \angle D = 180°$
2. $\Sigma\cos A = 0$ and $\Sigma\cos(180° - A) = 0$

- Let $AB = a$, $BC = b$, $CD = c$ and $DA = d$, then
- Area of cyclic quadrilateral

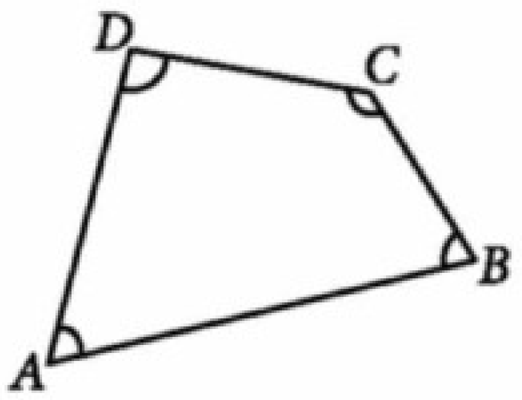
$$= \text{ar } \Delta ABC + \text{ar } \Delta CAD$$

$$=\frac{1}{2}ab\sin B+\frac{1}{2}cd\sin D=\frac{1}{2}(ab+cd)\sin D$$

$$=\sqrt{(s-a)(s-b)(s-c)(s-d)},\text{ where } s=\frac{a+b+c+d}{2}$$

$$= \Delta(\text{say})$$

- $\cos A=\frac{(a^2+d^2)-(b^2+c^2)}{2(ad+bc)}$ and similarly thers
- $(AC)^2 \cdot (BD)^2 = (ac + bd)^2$
  i.e., $AC \cdot BD = AB \cdot CD + BC \cdot AD$



- As A, B, C, D are angles of quadrilateral
  i.e., $\angle A + \angle B + \angle C + \angle D = 360°$

$$\Rightarrow \tan(A + B + C + D) = \tan 2\pi = 0$$

$$\Rightarrow \frac{\Sigma\tan A-\Sigma\tan A\tan B\tan C}{1-\Sigma\tan A\tan B+\tan A\tan B\tan C\tan D}=0$$

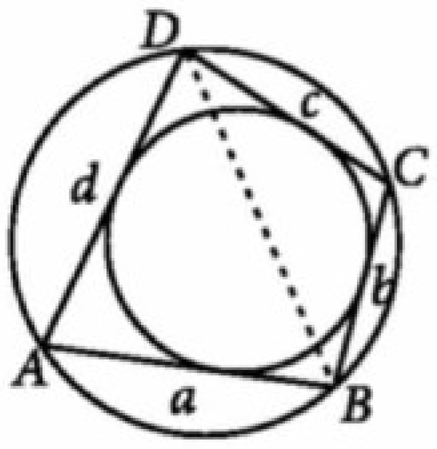
$$\Rightarrow \Sigma\tan A - \Sigma\tan A\tan B\tan C = 0$$

$$\Rightarrow \Sigma\tan A = \tan A\tan B\tan C\tan D \cdot \Sigma\cot A$$

$$\Rightarrow \frac{\Sigma\tan A}{\Sigma\cot A}=\Pi\tan A$$

(Product of all tangent angles)

If quadrilateral $ABCD$ in which $AB = a$, $BC = b$, $CD = c$ and $DA = d$ is such that one circle can be inscribed in it and another circle circumscribed about it then



1. $a+c=b+d$ (As circle is inscribed in the quadrilateral $ABCD$) or $a - d = b - c$
2. In ΔABD, $\cos A=\frac{a^2+d^2-(BD)^2}{2ad}$

$\Rightarrow (BD)^2 = a^2 + d^2 - 2ad\cos A$ ....(*)

3. In ΔBCD, $\cos C=\frac{b^2+c^2-(BD)^2}{2bc}$

$\Rightarrow (BD)^2 = b^2 + c^2 + 2bc\cos A$ ...(**)

$\therefore$ By (*) and (**), we get $\cos A=\frac{a^2+d^2-b^2-c^2}{2(bc+ad)}$,

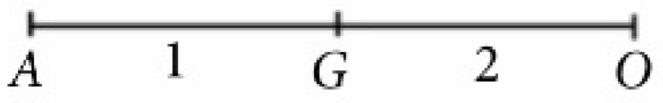
Now on using $(a - d)^2 = (b - c)^2$, we get

$a^2 + d^2 - b^2 - c^2 = 2(ad - bc)$ ...(***)

$\therefore \cos A=\frac{ad-bc}{ad+bc}$

Similarly for other cosine angles.

DISTANCES BETWEEN ORTHOCENTRE (O), CENTROID (G), CIRCUMCENTRE (A) AND INCENTRE (I)



(1) AO = Distance between circumcentre and orthocentre $=R\sqrt{1-8\cos A\cos B\cos C}$

(2) AG = Distance between centroid and circumcentre $=\frac{1}{3}(AO)$

(3) OG = Distance between orthocentre and centroid $=\frac{2}{3}(AO)$

(4) OI = Distance between orthocentre and incentre $=\sqrt{2r^2-4R^2\cos A\cos B\cos C}$

RULES OF EX-CIRCLES

Let r_1, r_2, r_3 are ex-radii and a, b, c are sides of triangle ABC then

- $r_1=\frac{\Delta}{s-a}=\sqrt{\frac{s(s-b)(s-c)}{s-a}}=s\tan\frac{A}{2}$,
  $r_2=\frac{\Delta}{s-b}=\sqrt{\frac{s(s-a)(s-c)}{s-b}}=s\tan\frac{B}{2}$,
  $r_3=\frac{\Delta}{s-c}=\sqrt{\frac{s(s-a)(s-b)}{s-c}}=s\tan\frac{C}{2}$
- $\frac{1}{r}=\frac{s}{\Delta}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}$
- $r_1 + r_2 + r_3 = 4R + r$

- $(r_1 - r)(r_2 - r)(r_3 - r) = 4Rr^2$
- $r^2 + r_1^2 + r_2^2 + r_3^2 = 16R^2 - (a^2 + b^2 + c^2)$
- $\left(\frac{1}{r}-\frac{1}{r_1}\right)\left(\frac{1}{r}-\frac{1}{r_2}\right)\left(\frac{1}{r}-\frac{1}{r_3}\right)=\frac{16R}{r^2(a+b+c)^2}$

  $=\frac{4R}{(rs)^2}=\frac{4R}{\Delta^2}=\frac{abc}{\Delta^3}$
- $a\cos A + b\cos B + c\cos C = \frac{r(a+b+c)}{R}$

  or $\frac{R(a\cos A+b\cos B+c\cos C)}{a+b+c}=r$

  or $\frac{\sin 2A+\sin 2B+\sin 2C}{\sin A+\sin B+\sin C}=\frac{2r}{R}$
- $\frac{1}{r^2}+\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^2}=\frac{a^2+b^2+c^2}{\Delta^2}$
- $a\cot A + b\cot B + c\cot C = 2(R + r)$
- If P_1, P_2, P_3 are perpendiculars (length) drawn from the vertices to opposite sides of the triangle then

(1) $\frac{1}{P_1}+\frac{1}{P_2}+\frac{1}{P_3}=\frac{1}{r}$ and $P_1P_2P_3=\frac{(abc)^2}{8R^3}$

(2) $\frac{\cos A}{P_1}+\frac{\cos B}{P_2}+\frac{\cos C}{P_3}=\frac{1}{R}$

(3) $\frac{1}{P_1^2}+\frac{1}{P_2^2}+\frac{1}{P_3^2}=\frac{a^2+b^2+c^2}{4\Delta^2}$

$=\frac{\cot A+\cot B+\cot C}{\Delta}$

PROBLEMS

Single Option Correct Type

1. Let a, b, c be the sides of a triangle whose area is Δ satisfy the condition $\Delta \le \frac{\lambda}{4}$, then the value of λ is

(a) $\sqrt{abc(a+b+c)}$ (b) $\sqrt{(a-b+c)(abc)}$

(c) $\sqrt{a+b+c}$ (d) abc

2. The minimum value of 'λ' for which the equation $\frac{a^2}{\sin x}+\frac{b^2}{1-\sin x}=\lambda$, $(a > b > 0)$ has at least one solution in $\left]0,\frac{\pi}{2}\right[$ is

(a) $a^2 + b^2$ (b) $a^2 - b^2$ (c) $(a + b)^2$ (d) $(a - b)^2$

3. Let x be in radian such that $0 < x < \frac{\pi}{2}$. If $\sin(3\sin x) = \cos(3\cos x)$, then $\tan x + \cot x$ can be written as $\frac{a}{\pi^b - c}$ where a, b, c are positive integers, then value of $\left(\frac{a+b+c}{22}\right)$ is

(a) 2 (b) 3 (c) 4 (d) 5

4. If $f(x)=\tan\frac{3\pi}{2}[x]+\cos^2\pi[x]+e^{\{x/4\}}$, where $[\cdot]$ represents the greatest integer function and $\{\}$ represents fractional part, then

(a) $f(x)$ is non-periodic function

(b) $f(x)$ is periodic but period is not defined

(c) $f(x)$ is periodic with period 6

(d) $f(x)$ is periodic with period 12

5. The number of solution(s) of $\sin^4 x + \cos^3 x \ge 1$ in $]0, 2\pi[$ is

(a) 1 (b) 2

(c) 3 (d) none of these

6. If $e^{\{\sin^2 x+\sin^4 x+\ldots\infty\}\log_e 2}$ satisfies the equation $x^2 - 9x + 8 = 0$, then the value of $\frac{\cos x}{\sin x+\cos x}$, $x\in\left(0,\frac{\pi}{2}\right)$ is

(a) $\frac{\sqrt{3}-1}{2}$ (b) $\sqrt{3}-1$ (c) $\sqrt{3}+1$ (d) $\frac{\sqrt{3}+1}{2}$

7. If in ΔABC, the line joining the circumcentre and incentre is parallel to BC, then

(a) $r = R\sin A$ (b) $r = R\cos A$

(c) $R = r\sin A$ (d) $R = r\cos A$

8. In a triangle ABC, sides a, b, c are in A.P., then $\sin A \sin B \cos^2 C/2$, $\sin A \sin C \cos^2 B/2$ and $\sin B \sin C \cos^2 A/2$ are in

(a) H.P. (b) G.P.

(c) A.P. (d) none of these

9. A square is inscribed in an equilateral triangle of side 5 units such that two of its vertices lies on one side, one each on other two sides except the square part, therefore there are 3 triangular parts. The sum of the circum-radii of these three triangles is

(a) $8(2-\sqrt{3})$ (b) $8(2+\sqrt{3})$

(c) $15(2+\sqrt{3})$ (d) $15(2-\sqrt{3})$

10. In a triangle ABC, if $A=\frac{\pi}{7}, B=\frac{2\pi}{7}, C=\frac{4\pi}{7}$, then $\frac{a^2+b^2+c^2}{R^2}$ equals

(a) 3 (b) 5 (c) 7 (d) 9

11. In a triangle ABC, $a : b : c = 4 : 5 : 6$. If the ratio of the radius of circumcircle to that of the incircle is $\frac{\alpha}{\beta}$ then $\alpha - \beta$ equals

(a) 3 (b) 5 (c) 7 (d) 9

More Than One Option(s) Correct Type

12. $\sin\theta - \sqrt{3}\cos\theta = 8x - x^2 - 18$, $0 \le \theta \le 4\pi$, $x \in R$ hold for

(a) less than two values of x and less than three values of θ

(b) two pair of values of (x, θ)

(c) one value of x and two values of θ

(d) two values of x and two values of θ

13. In a triangle if $2b^2c^2 + 2c^2a^2 = a^4 + b^4 + c^4$ then angle C equal to

(a) 75° (b) 45° (c) 135° (d) 60°

14. In a triangle if $\Delta = a^2 - (b - c)^2$ then which of the following is/are correct?

(a) $\tan\frac{A}{2} = \frac{1}{4}$ (b) $\operatorname{cosec} A = \frac{17}{8}$

(c) $\cot A = \frac{15}{8}$ (d) $\sin\frac{A}{2} = \frac{1}{\sqrt{17}}$

15. The number of real roots of the equation $\cos^7 x + \sin^4 x = 1$ in the interval $(-\pi, \pi)$ is/are less than

(a) 4 (b) 5 (c) 6 (d) 7

Comprehension Type

Paragraph for Q. No. 16 to 20

Let $\cos\frac{2\pi}{7}, \cos\frac{4\pi}{7}, \cos\frac{6\pi}{7}$ are roots of the equation $8x^3 + 4x^2 - 4x - 1 = 0$.

16. The value of $\sec\frac{2\pi}{7} + \sec\frac{4\pi}{7} + \sec\frac{6\pi}{7}$ is

(a) -4 (b) $1/4$

(c) 4 (d) none of these

17. The value of $\sin\frac{\pi}{7}\sin\frac{2\pi}{7}\sin\frac{3\pi}{7}$ is

(a) $\frac{\sqrt{7}}{4}$ (b) $\frac{\sqrt{7}}{8}$ (c) $\frac{1}{8}$ (d) $-\frac{1}{8}$

18. The value of

$\sec\frac{2\pi}{7}\sec\frac{4\pi}{7} + \sec\frac{4\pi}{7}\sec\frac{6\pi}{7} + \sec\frac{2\pi}{7}\sec\frac{6\pi}{7}$ is

(a) 8 (b) $\frac{\sqrt{7}}{8}$ (c) $\frac{1}{8}$ (d) -4

19. The value of $\sec\frac{2\pi}{7}\sec\frac{4\pi}{7}\sec\frac{6\pi}{7}$ is

(a) 1/8 (b) $-1/8$ (c) 8 (d) -8

20. The equation whose roots are $\sec^2\frac{2\pi}{7}, \sec^2\frac{4\pi}{7}, \sec^2\frac{6\pi}{7}$ is

(a) $x^3 + 24x^2 - 80x + 64 = 0$

(b) $x^3 - 24x^2 + 80x - 64 = 0$

(c) $x^3 - 24x^2 - 80x - 64 = 0$

(d) $x^3 + 80x^2 - 24x - 64 = 0$

Matrix-Match Type

21. Match the following :

	Column I		Column II
P.	$\cot 82\frac{1}{2}^\circ =$	1.	$\cot 22\frac{1}{2}^\circ$
Q.	$\cot 15^\circ =$	2.	$\sqrt{2}+1$
R.	$\tan 82\frac{1}{2}^\circ =$	3.	$2+\sqrt{3}$
S.	$\tan 67\frac{1}{2}^\circ =$	4.	$(\sqrt{3}+\sqrt{2})(\sqrt{2}+1)$
		5.	$\cot 7\frac{1}{2}^\circ$
		6.	$(\sqrt{3}-\sqrt{2})(\sqrt{2}-1)$

Numerical Value Type

22. In a triangle, L and M are midpoints of the sides AB and AC respectively. Let 'O' is the circumcentre of the ΔABC, then $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta OLM)}\cot B\cot C$ equals

23. If K be the difference between the greatest and least values of $f(x) = \cos x + \frac{\cos 2x}{2} - \frac{\cos 3x}{3}$, then $4K$ equals

24. The number of real values of x, satisfying the equation $[x]^2 - 5[x] + 6 - \sin x = 0$, ([.] denotes the greatest integer function) is

25. The number of solutions of the equation $\tan^2 x + \cot^2 x + \operatorname{cosec}^2 x - 4\cos^2 x = 2$ in $[0, \pi]$ is

26. In a triangle ABC, $a = 7$, $b = 6$ and $\cos(A - B) = \frac{251}{256}$, then the third side of the triangle must be

27. If $\cos\theta + \cos^2\theta + \cos^3\theta = 1$ and $\sin^6\theta - 4\sin^4\theta + 8\sin^2\theta = k^2$ then k must be

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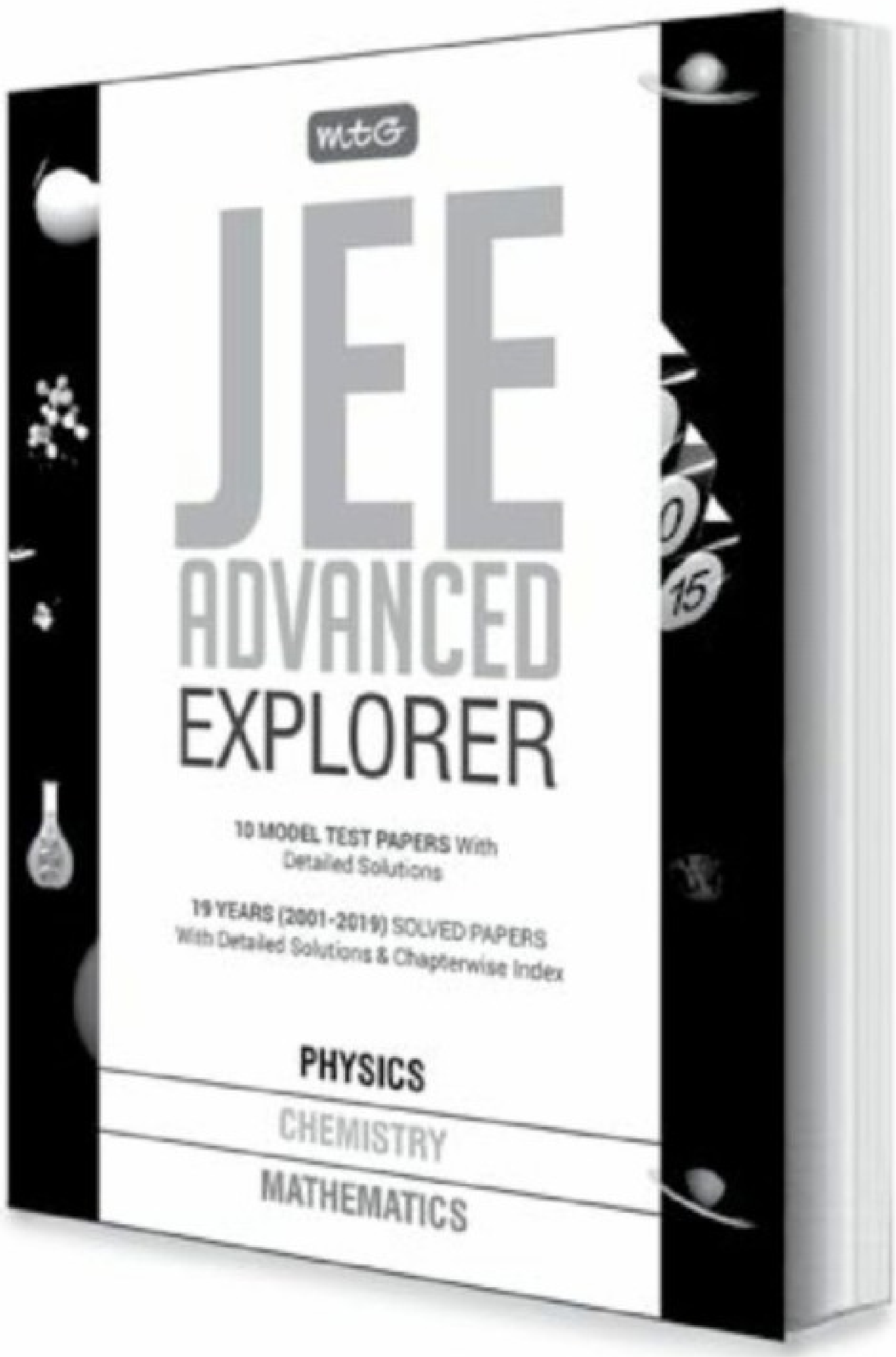
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28. If l_1, l_2, l_3 are the altitudes of a triangle ABC which circumscribes a circle of diameter 14/3 units, then the least value of $l_1 + l_2 + l_3$ must be

SOLUTIONS

1. (a)

2. (c) : As $\lambda = \dfrac{a^2}{\sin x} + \dfrac{b^2}{1-\sin x}$

$$\therefore \frac{d\lambda}{dx} = \cos x\left[\frac{-a^2}{\sin^2 x} + \frac{b^2}{(1-\sin x)^2}\right]$$

$$\Rightarrow \frac{d\lambda}{dx} = \frac{\cos x\,[(b^2-a^2)\sin^2 x + 2a^2\sin x - a^2]}{\sin^2 x\,(1-\sin x)^2}$$

For minimum value of λ, $\dfrac{d\lambda}{dx} = 0$

$$\Rightarrow \cos x = 0,\ \sin x = \frac{a}{a-b},\ \sin x = \frac{a}{a+b}$$

But $\sin x = \dfrac{a}{a+b}$ is only possible value.

$\therefore$ Minimum value of $\lambda = \dfrac{a^2(a+b)}{a} + \dfrac{b^2(a+b)}{b}$

$= (a + b)^2$

3. (d) : $\because \sin(3\sin x) = \cos(3\cos x)$

$$\Rightarrow \sin(3\sin x) = \sin\left(\frac{\pi}{2} - 3\cos x\right) \Rightarrow \sin x + \cos x = \frac{\pi}{6}$$

$$\Rightarrow 1 + \sin 2x = \frac{\pi^2}{36} \quad \Rightarrow \sin 2x = \frac{\pi^2 - 36}{36}$$

Now, $\tan x + \cot x = \dfrac{2}{\sin 2x} = \dfrac{2\times 36}{\pi^2 - 36} = \dfrac{72}{\pi^2 - 36} = \dfrac{a}{\pi^b - c}$

$\therefore$ $a = 72$, $b = 2$ and $c = 36$

$$\therefore \frac{a+b+c}{22} = 5$$

4. (d) : Period of $e^{\{x/4\}}$ is 4

For period of $\tan\dfrac{3\pi}{2}[x]$, we have

$$\tan\frac{3\pi}{2}[x+t] = \tan\frac{3\pi}{2}[x] \Rightarrow \frac{3\pi}{2}[x+t] = n\pi + \frac{3\pi}{2}[x]$$

$\Rightarrow$ $3([x + t] - [x]) = 2n$ $\Rightarrow$ $n = 3$, $t = 2$

and $\cos^2\pi[x + t] = \cos^2\pi[x]$

$\Rightarrow$ $\pi[x + t] = n\pi + \pi[x]$

(as $\cos^2\alpha = \cos^2\beta \Rightarrow \alpha = n\pi + \beta$)

$\Rightarrow$ $[x + t] - [x] = n$ $\Rightarrow$ $n = 1$, $t = 1$

$\therefore$ L.C.M. of 1, 2, 3, 4 is 12.

5. (c) : We know that $\sin^2 x + \cos^2 x = 1$, $-1 \le \sin x, \cos x \le 1$

Now, $\sin^4 x + \cos^3 x \nsucc 1$, we have $\sin^4 x + \cos^3 x = \pm 1$, which is possible only if $\sin x = 1$ and $\cos x = 0$ or $\sin x = 0$ and $\cos x = -1$

$$\Rightarrow x = (4n+1)\frac{\pi}{2},\ x = (2n+1)\frac{\pi}{2},$$

or $x = n\pi$, $x = 2n\pi$

Now, we need solution in $(0, 2\pi)$

$\therefore$ $x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$ or $x = \pi$ as $x \ne 0, 2\pi$

$\therefore$ Number of solutions are 3 *i.e.*, $\dfrac{\pi}{2}, \pi, \dfrac{3\pi}{2} \in (0, 2\pi)$

6. (a) : $e^{\{\sin^2 x + \sin^4 x + \ldots\infty\}\log_e 2}$

$$= e^{\left(\frac{\sin^2 x}{1-\sin^2 x}\right)\log_e 2} = 2^{\tan^2 x}$$

Now, $x^2 - 9x + 8 = 0$

$\Rightarrow$ $(x - 1)(x - 8) = 0$ $\Rightarrow$ $x = 1$ or 8

$\therefore$ $2^{\tan^2 x} = 1$ or $2^{\tan^2 x} = 2^3$

$\Rightarrow$ $\tan^2 x = 0$ or $\tan^2 x = 3 = \tan^2 60°$

$$\Rightarrow x = 0 \text{ or } x = \pm\frac{\pi}{3}$$

But only $x = \dfrac{\pi}{3} \in \left(0, \dfrac{\pi}{2}\right)$. $\therefore$ $x = \dfrac{\pi}{3}$

$$\therefore \frac{\cos x}{\sin x + \cos x} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2} + \frac{1}{2}} = \frac{\sqrt{3}-1}{2}$$

7. (b) : Let S be the circumcentre and I be the incentre of ΔABC.

As $SI \parallel BC$, $IN = SM$

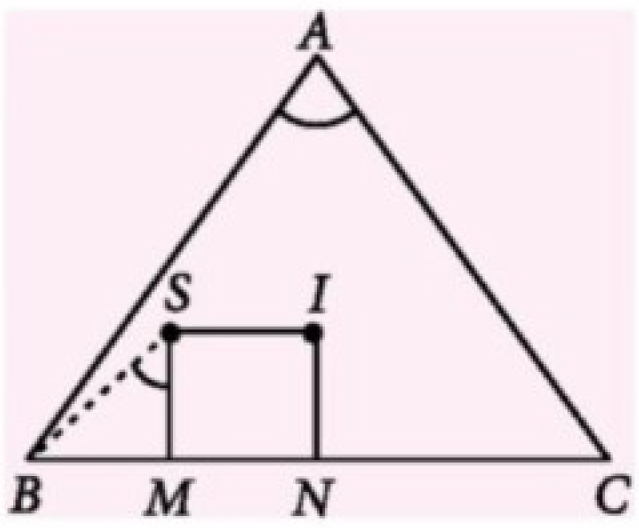
$= r$ (in -radius)

Also, $SB = R$ and $\angle BSM = \angle A$

From ΔSBM,

$$\cos A = \frac{SM}{SB} = \frac{r}{R}$$

$\Rightarrow$ $r = R\cos A$



8. (c) : As a, b, c are in A.P. $\Rightarrow$ $2b = a + c$

$$\text{Now, } \sin A \sin B \cos^2\frac{C}{2} = \frac{a}{2R}\cdot\frac{b}{2R}\cdot\frac{s(s-c)}{ab} = \frac{s(s-c)}{4R^2} \quad \text{... (i)}$$

$$\text{Similarly, } \sin A \sin C \cos^2\frac{B}{2} = \frac{a}{2R}\cdot\frac{c}{2R}\cdot\frac{s(s-b)}{ac} = \frac{s(s-b)}{4R^2} \quad \text{...(ii)}$$

$$\text{and } \sin B \sin C \cos^2\frac{A}{2} = \frac{b}{2R}\cdot\frac{c}{2R}\cdot\frac{s(s-a)}{bc} = \frac{s(s-a)}{4R^2} \quad \text{... (iii)}$$

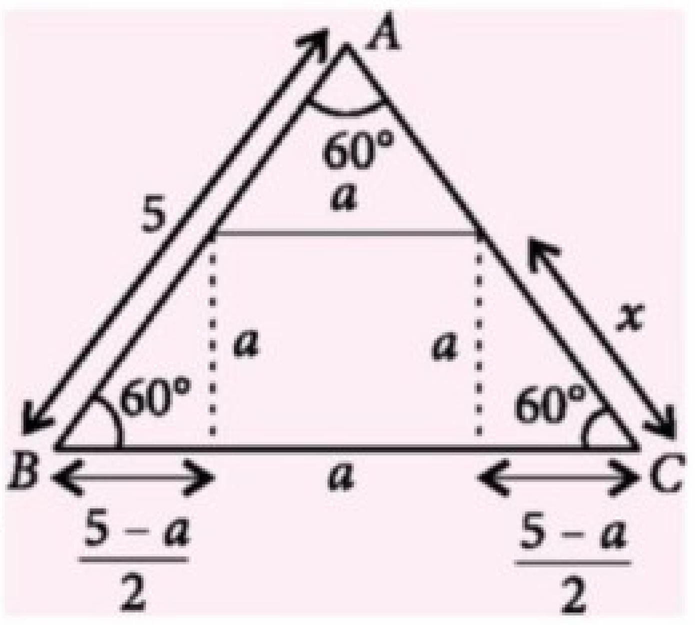
Adding (i) and (iii), we get

$$\frac{s(s-c)+s(s-a)}{4R^2} = \frac{s[2s-(c+a)]}{4R^2} = \frac{s(2s-2b)}{4R^2} = \frac{2s(s-b)}{4R^2}$$

$\therefore$ (i), (ii) and (iii) are in A.P.

9. (d) : Triangle is equilateral with side 5 units. Let each side of square be 'a' unit and $AB = BC = CA = 5$ units.

$$\text{Now, } \tan 60° = \frac{a}{\frac{5-a}{2}} \Rightarrow a = \frac{5\sqrt{3}}{2+\sqrt{3}} = 5\sqrt{3}(2-\sqrt{3})$$



$$\text{Again, } \sin 60° = \frac{a}{x}$$

$$\Rightarrow \quad x = \frac{2a}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{x}{2} = \frac{a}{\sqrt{3}} \text{ (circumradius)}$$

$$\text{Now, } \frac{a}{\sin 60°} = 2R \Rightarrow R = \frac{a}{\sqrt{3}}$$

As two triangular parts are congruent.

∴ Their circum-radii are equal.

Thus, circum-radii are $\frac{a}{\sqrt{3}}, \frac{a}{\sqrt{3}}, \frac{a}{\sqrt{3}}$.

$$\therefore \quad \text{Sum of all circum-radii} = 3 \times \frac{a}{\sqrt{3}} = a\sqrt{3}$$

$$= \sqrt{3}(5\sqrt{3})(2-\sqrt{3}) = 15(2-\sqrt{3})$$

10. (c)

11. (d) : ∵ $a : b : c = 4 : 5 : 6$, let $a = 4k$, $b = 5k$, $c = 6k$

$$\therefore \quad s = \frac{15}{2}k, \; s-a = \frac{7k}{2}, s-b = \frac{5k}{2}, s-c = \frac{3}{2}k$$

$$\text{And } \Delta = \sqrt{s(s-a)(s-b)(s-c)} = \frac{15\sqrt{7}}{4}k^2$$

$$\therefore \quad \text{In-radius of triangle, } r = \frac{\Delta}{s} = 15\sqrt{7}\frac{k^2}{4} \times \frac{2}{15k} = \frac{\sqrt{7}}{2}k$$

$$\text{Circumradius of triangle, } R = \frac{abc}{4\Delta} = \frac{8}{\sqrt{7}}k$$

$$\therefore \quad r : R = \frac{\sqrt{7}}{2}k : \frac{8}{\sqrt{7}}k = 7 : 16$$

$$\Rightarrow \quad \frac{R}{r} = \frac{16}{7} = \frac{\alpha}{\beta} \Rightarrow \alpha - \beta = 16 - 7 = 9$$

12. (a, b, c) : ∵ $\sin\theta - \sqrt{3}\cos\theta = 8x - x^2 - 18$, $0 \le \theta \le 4\pi$

$$\Rightarrow \sin\left(\theta - \frac{\pi}{3}\right) = \frac{8x - x^2 - 18}{2}$$

$$\because \quad -1 \le \sin\left(\theta - \frac{\pi}{3}\right) \le 1$$

⇒ $-2 \le 8x - x^2 - 18 \le 2$

⇒ $0 \le 8x - x^2 - 16 \le 4$ ⇒ $0 \le -(x-4)^2 \le 4$

Now, $x = 4$ is the only value which satisfies the equality ⇒ $(x-4)^2 \le 0$ which is possible only when $x - 4 = 0$ *i.e.*, $x = 4$

and $(x-4)^2 \ge -4$ and $(x-4)^2 + 4 \ge 0$ which is always true.

∴ At $x = 4$, we have

$$\sin\left(\theta - \frac{\pi}{3}\right) = -1 \Rightarrow \begin{cases} \theta - \frac{\pi}{3} = 2n\pi - \frac{\pi}{2} \Rightarrow \theta = 2n\pi - \frac{\pi}{6} \\ \text{or } \theta = n\pi + (-1)^{n+1}\frac{\pi}{2} + \frac{\pi}{3} \end{cases}$$

If $n = 0$ then $\theta \notin [0, 4\pi]$

$$n = 1, 2, 3, 4 \text{ then } \theta = \frac{11\pi}{6}, \frac{23\pi}{6}$$

$$\therefore \quad (x, \theta) = \left(4, \frac{11\pi}{6}\right), \left(4, \frac{23\pi}{6}\right)$$

So, for one value of x there are two values of θ.

13. (b, c)

14. (a, b, c, d) : ∵ $\Delta = a^2 - (b-c)^2$

⇒ $\Delta = (a + b - c)(a - b + c) = 4(s - c)(s - b)$

$$\Rightarrow \quad \sqrt{s(s-a)(s-b)(s-c)} = 4(s-c)(s-b)$$

$$\Rightarrow \quad \sqrt{\frac{s(s-a)}{(s-b)(s-c)}} = 4 \quad \therefore \quad \tan\frac{A}{2} = \frac{1}{4}$$

$$\Rightarrow \quad \sin\frac{A}{2} = \frac{1}{\sqrt{17}}$$

$$\text{Again, } \tan A = \frac{2\tan A/2}{1 - \tan^2(A/2)} = \frac{2 \cdot \frac{1}{4}}{1 - \frac{1}{16}} = \frac{1}{2} \cdot \frac{16}{15} = \frac{8}{15}$$

$$\Rightarrow \cot A = \frac{15}{8} \text{ and } \operatorname{cosec} A = \frac{17}{8}$$

15. (a, b, c, d) : ∵ $\cos^7 x \le \cos^2 x$ ...(i)

and $\sin^4 x \le \sin^2 x$ ...(ii)

Adding (i) and (ii) we get

$\cos^7 x + \sin^4 x \le \cos^2 x + \sin^2 x = 1$

Equality holds only if

$\cos^7 x = \cos^2 x$ and $\sin^4 x = \sin^2 x$

⇒ $\cos^2 x(1 - \cos^5 x) = 0$ and $\sin^2 x(1 - \sin^2 x) = 0$

⇒ $\cos^2 x = 0$ or $\cos^5 x = 1$ and $\sin^2 x = 0$ or $\sin^2 x = 1$

⇒ $\cos x = 0$ or $\cos x = 1$ and $\sin x = 0$ or $\sin x = \pm 1$

As $x \in (-\pi, \pi)$, ∴ $x = -\frac{\pi}{2}, 0, \frac{\pi}{2}$ in both the cases.

∴ Total number of solutions is 3.

16. (a) **17. (b)** **18. (d)** **19. (c)** **20. (b)**

(16 – 20) : As $\cos\frac{2\pi}{7}, \cos\frac{4\pi}{7}, \cos\frac{6\pi}{7}$ are roots of the equation

$8x^3 + 4x^2 - 4x - 1 = 0$ ...(i)

∴ The equation whose roots are reciprocal of the roots of (i) is obtained by replacing x by $1/x$ in (i).

Now, equation whose roots are $\sec\frac{2\pi}{7}, \sec\frac{4\pi}{7}, \sec\frac{6\pi}{7}$ is

$x^3 + 4x^2 - 4x - 8 = 0$ ...(ii)

$$\therefore\ \sec\frac{2\pi}{7}+\sec\frac{4\pi}{7}+\sec\frac{6\pi}{7}=\frac{-\text{coeff. of } x^2}{\text{coeff. of } x^3}=-4$$

and $\sec\frac{2\pi}{7}\ \sec\frac{4\pi}{7}+\sec\frac{4\pi}{7}\sec\frac{6\pi}{7}+\sec\frac{2\pi}{7}\ \sec\frac{6\pi}{7}$

$$=\frac{\text{coeff. of } x}{\text{coeff. of } x^3}=-4$$

Also, $\sec\frac{2\pi}{7}\cdot\sec\frac{4\pi}{7}\cdot\sec\frac{6\pi}{7}=8$

or $\cos\frac{2\pi}{7}\cdot\cos\frac{4\pi}{7}\cdot\cos\frac{6\pi}{7}=\frac{1}{8}$

Now, again by (i), we have, $8x^3 + 4x^2 - 4x - 1$

$$= 8\left(x-\cos\frac{2\pi}{7}\right)\left(x-\cos\frac{4\pi}{7}\right)\left(x-\cos\frac{6\pi}{7}\right) \quad ...(iii)$$

Putting $x = 1$ in (iii), we get

$$8+4-4-1=8\left(1-\cos\frac{2\pi}{7}\right)\left(1-\cos\frac{4\pi}{7}\right)\left(1-\cos\frac{6\pi}{7}\right)$$

$$\Rightarrow\ 7=64\left(\sin\frac{\pi}{7}\sin\frac{2\pi}{7}\sin\frac{3\pi}{7}\right)^2$$

$$\Rightarrow\ \sin\frac{\pi}{7}\sin\frac{2\pi}{7}\sin\frac{3\pi}{7}=\frac{\sqrt{7}}{8}$$

Again to obtain equation, whose roots are

$$\sec^2\frac{2\pi}{7}, \sec^2\frac{4\pi}{7} \text{ and } \sec^2\frac{6\pi}{7}$$

replace x by $\frac{1}{\sqrt{x}}$ in the given equation

$$\frac{8}{x\sqrt{x}}+\frac{4}{x}-\frac{4}{\sqrt{x}}-1=0\ \Rightarrow\ \frac{1}{\sqrt{x}}(8-4x)=(x-4)$$

$\Rightarrow\ (8-4x)^2=x(x-4)^2$

$\Rightarrow\ 64 - 64x + 16x^2 = x(x^2 - 8x + 16)$

$\Rightarrow\ x^3 - 24x^2 + 80x - 64 = 0$

21. P → 6; Q → 3; R → 4, 5; S → 1, 2

(P) $\cot 82\frac{1}{2}^\circ=\tan 7\frac{1}{2}^\circ=\dfrac{\sin 7\frac{1}{2}^\circ}{\cos 7\frac{1}{2}^\circ}$

$$=\frac{2\sin^2 7\frac{1}{2}^\circ}{2\sin 7\frac{1}{2}^\circ\ \cos 7\frac{1}{2}^\circ}=\frac{1-\cos 15^\circ}{\sin 15^\circ}=\frac{1-\dfrac{\sqrt{3}+1}{2\sqrt{2}}}{\dfrac{\sqrt{3}-1}{2\sqrt{2}}}$$

$$=\frac{(2\sqrt{2}-\sqrt{3}-1)(\sqrt{3}+1)}{2}$$

$$=\sqrt{6}-\sqrt{3}+\sqrt{2}-2=(\sqrt{3}-\sqrt{2})(\sqrt{2}-1)$$

(Q) $\cot 15^\circ=\dfrac{\cos 15^\circ}{\sin 15^\circ}=\dfrac{2\cos^2 15^\circ}{2\sin 15^\circ\ \cos 15^\circ}$

$$=\frac{1+\cos 30^\circ}{\sin 30^\circ}=2+\sqrt{3}$$

(R) $\tan 82\frac{1}{2}^\circ=\cot 7\frac{1}{2}^\circ=\dfrac{\cos 7\frac{1}{2}^\circ}{\sin 7\frac{1}{2}^\circ}=\dfrac{2\cos^2 7\frac{1}{2}^\circ}{2\sin 7\frac{1}{2}^\circ\ \cos 7\frac{1}{2}^\circ}$

$$=\frac{1+\cos 15^\circ}{\sin 15^\circ}=\frac{1+\dfrac{\sqrt{3}+1}{2\sqrt{2}}}{\dfrac{\sqrt{3}-1}{2\sqrt{2}}}=\frac{2\sqrt{2}+\sqrt{3}+1}{\sqrt{3}-1}$$

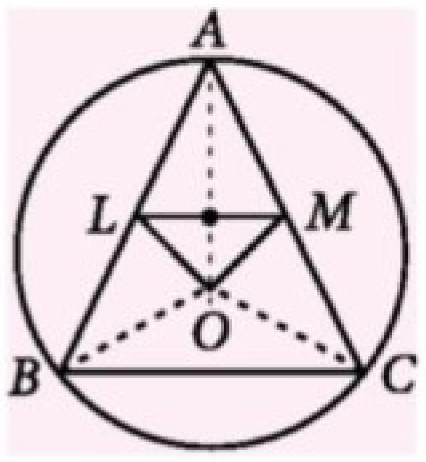
$$=\sqrt{6}+\sqrt{3}+\sqrt{2}+2$$

(S) $\tan 67\frac{1}{2}^\circ=\cot 22\frac{1}{2}^\circ=\dfrac{\cos 22\frac{1}{2}^\circ}{\sin 22\frac{1}{2}^\circ}$

$$=\frac{2\cos^2 22\frac{1}{2}^\circ}{2\sin 22\frac{1}{2}^\circ\ \cos 22\frac{1}{2}^\circ}=\frac{1+\cos 45^\circ}{\sin 45^\circ}=\sqrt{2}+1$$

22. (4) : Let $AB = c$, $BC = a$ and $AC = b$.

As L, M are midpoints of AB and AC respectively.



$$\therefore\ LM=\frac{1}{2}BC=\frac{a}{2}$$

Also, $OA = OB = OC = R$ = circumradius of ΔABC and $\angle AOM = \angle B$ and $OM = OA\cos B = R\cos B$, now $\angle AML = \angle C$

Since, $LM \parallel BC$ as L, M are midpoints of AB and AC respectively.

$$\therefore\ \angle LMO=\frac{\pi}{2}-\angle C$$

$$\therefore\ \text{Area of triangle } OLM\ =\frac{1}{2}LM\cdot OM\sin\angle LMO$$

$$=\frac{1}{2}\frac{BC}{2}(R\cos B)\cos C=\frac{aR\cos B\cos C}{4}$$

We know that,

$$\text{area of } \Delta ABC=\frac{1}{2}ab\sin C=\frac{a}{2}(2R\sin B\sin C)$$

$= aR\sin B\sin C$

$$\text{Thus, }\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta OLM)}=\frac{aR\sin B\sin C}{\frac{aR}{4}\cos B\cos C}=4\tan B\tan C$$

$$\Rightarrow\ \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta OLM)}\cot B\cot C=4$$

23. (9) : The given function is periodic with period 2π so the difference between the greatest and least value will be the difference of these values in the interval $[0, 2\pi]$.

Now, $f(x)=\cos x+\frac{\cos 2x}{2}-\frac{\cos 3x}{3}$

$\Rightarrow\ f'(x) = -(\sin x + \sin 2x - \sin 3x)$

$=-4\sin x\ \sin\frac{3x}{2}\ \sin\frac{x}{2}$

Hence, the critical points are $0, \frac{2\pi}{3}, \pi$ and 2π.

$\therefore\ f(0)=1+\frac{1}{2}-\frac{1}{3}=\frac{7}{6},\ f\left(\frac{2\pi}{3}\right)=\frac{-13}{12},$

$f(\pi)=-\frac{1}{6}, f(2\pi)=\frac{7}{6}$

$\therefore$ Greatest value and least values of $f(x)$ are respectively $\frac{7}{6}$ and $-\frac{13}{12}$.

$\therefore\ K=\frac{7}{6}+\frac{13}{12}=\frac{9}{4}\ \Rightarrow\ 4K=9$

24. (1) : Solve the two equations $\sin x = 0$ and $[x]^2 - 5[x] + 6 = 0$

The equation $[x]^2 - 5[x] + 6 = 0 \Rightarrow [x] = 3, 2$

the equation $\sin x = 0\ \Rightarrow x = 0, \pi, 2\pi, ..., n\pi$

$\therefore\ [x] = [0], [\pi], [2\pi], ..., [n\pi] = 0, 3, 6, 9 .....$

Thus, there is only one common root $[x] = 3$

$\therefore$ Number of common root = 1

25. (2) : From the given equation

$\tan^2 x + \cot^2 x + 1 + \cot^2 x + 4\sin^2 x = 6$

$\Rightarrow\ \tan^2 x+2\cot^2 x+\frac{4\sin^2 x}{\sin^2 x+\cos^2 x}=5$

$\Rightarrow\ \tan^2 x+2\cot^2 x+\frac{4\tan^2 x}{1+\tan^2 x}=5$

$\Rightarrow\ t^2+\frac{2}{t^2}+\frac{4t^2}{1+t^2}=5$ (By putting $\tan x = t$)

$\Rightarrow\ t^6 - 3t^2 + 2 = 0$

$\Rightarrow\ (t^2 - 1)(t^4 + t^2 - 2) = 0 \Rightarrow (t^2 - 1)^2(t^2 + 2) = 0$

$\Rightarrow\ t^2 = 1 \Rightarrow \tan x = \pm 1$

$\Rightarrow\ x=\frac{\pi}{4}, \frac{3\pi}{4}$ as $x \in [0, \pi]$ only two values are possible and $t^4 + t^2 - 2 = 0$ rejected as either values are repeated in $[0, \pi]$ or does not belongs to $[0, \pi]$.

$\therefore$ Only two values are possible in $[0, \pi]$.

26. (8) : $\because\ \cos(A-B)=\frac{1-\tan^2\left(\frac{A-B}{2}\right)}{1+\tan^2\left(\frac{A-B}{2}\right)}$

$\Rightarrow\ \frac{251}{256}=\frac{1-\tan^2\left(\frac{A-B}{2}\right)}{1+\tan^2\left(\frac{A-B}{2}\right)}$

$\Rightarrow\ \tan\left(\frac{A-B}{2}\right)=\sqrt{\frac{5}{507}}\ \Rightarrow\ \frac{a-b}{a+b}\cot\frac{C}{2}=\sqrt{\frac{5}{507}}$

$\Rightarrow\ \cot\frac{C}{2}=\frac{13\sqrt{5}}{\sqrt{507}}\ \Rightarrow\ \tan\frac{C}{2}=\frac{\sqrt{507}}{13\sqrt{5}}$

Again, $\cos C=\frac{1-\tan^2 C/2}{1+\tan^2 C/2}=\frac{1-\frac{507}{845}}{1+\frac{507}{845}}=\frac{338}{1352}=\frac{1}{4}$

$\Rightarrow\ \frac{a^2+b^2-c^2}{2ab}=\frac{1}{4}\ \Rightarrow\ \frac{49+36-c^2}{2\cdot 7\cdot 6}=\frac{1}{4}$

$\Rightarrow\ c^2 = 64\ \Rightarrow\ c = 8$

27. (2)

28. (21) : As l_1, l_2, l_3 are altitudes of the triangle ABC

$\therefore\ l_1=\frac{2\Delta}{a},\ l_2=\frac{2\Delta}{b},\ l_3=\frac{2\Delta}{c}$

$\therefore\ l_1 + l_2 + l_3 \geq 3(l_1 l_2 l_3)^{1/3}$ (Using A.M. ≥ G.M.)

$=3\left\{\frac{(2\Delta)^3}{abc}\right\}^{1/3}=6\Delta\left(\frac{1}{abc}\right)^{1/3}$

$=6\ r\cdot s\left(\frac{1}{abc}\right)^{1/3}=6r\left(\frac{a+b+c}{2}\right)\left(\frac{1}{abc}\right)^{1/3}$

$=9r\left(\frac{a+b+c}{3}\right)\left(\frac{1}{abc}\right)^{1/3}$

$\geq 9\cdot r=9\times\frac{14}{3}\times\frac{1}{2}$ ($\because$ Diameter = 2 Radius)

$= 21$

$\Rightarrow\ l_1 + l_2 + l_3 \geq 21$

$\therefore$ Least value of $l_1 + l_2 + l_3$ is 21 units.

◆

Monthly Test Drive – 6 CLASS XII ANSWER KEY

1.	(b)	**2.**	(b)	**3.**	(c)	**4.**	(b)
5.	(c)	**6.**	(a)	**7.**	(a, c, d)	**8.**	(b, d)
9.	(b, c)	**10.**	(a, d)	**11.**	(a, c)	**12.**	(a, b, c)
13.	(a, c)	**14.**	(b)	**15.**	(a)	**16.**	(b)
17.	(521)	**18.**	(1)	**19.**	(0.31)	**20.**	(6561)

Challenging PROBLEMS ON Differential Equations

1. The family of curves passing through (0, 0) and satisfying the differential equation $\frac{y_2}{y_1}=1$ (where $y_n = d^ny/dx^n$) is

(a) $y = k$ (b) $y = kx$

(c) $y = k(e^x + 1)$ (d) $y = k(e^x - 1)$

2. The order of the differential equation of all tangent lines to the parabola $y = x^2$ is

(a) 1 (b) 2 (c) 3 (d) 4

3. If $y_1(x)$ is a solution of the differential equation $dy/dx - f(x)y = 0$, then a solution of the differential equation $\frac{dy}{dx}+f(x)y=r(x)$ is

(a) $\frac{1}{y_1(x)}\int r(x)y_1(x)dx$ (b) $y_1(x)\int\frac{r(x)}{y_1(x)}dx$

(c) $\int r(x)y_1(x)dx$ (d) None of these

4. The general differential equation of all parabola's is (where $y_n=\frac{d^ny}{dx^n}$)

(a) $5y_3^2 = 3y_2y_4$ (b) $5y_3^2 = 3y_2y_5$

(c) $5y_3^2 = y_2y_4$ (d) $5y_3^2 = y_2y_5$

5. The solution of the differential equation $y_1y_3 = 3y_2^2$ is

(a) $x = A_1y^2 + A_2y + A_3$ (b) $x = A_1y + A_2$

(c) $x = A_1y^2 + A_2y$ (d) None of these

6. The solution of the differential equation $(x^2 \sin^3 y - y^2 \cos x)dx + (x^3 \cos y \sin^2 y - 2y \sin x)dy = 0$ is

(a) $x^3 \sin^3 y = 3y^2 \sin x + C$

(b) $x^3 \sin^3 y + 3y^2 \sin x = C$

(c) $x^2 \sin^3 y + y^3 \sin x = C$ (d) $2x^2 \sin y + y^2 \sin x = C$

7. Solution of the differential equation $2y\sin x\frac{dy}{dx}=2\sin x\cos x-y^2\cos x$ satisfying $y(\pi/2)=1$ is given by

(a) $y^2 = \sin x$ (b) $y = \sin^2 x$

(c) $y^2 = \cos x + 1$ (d) $y^2 \sin x = 4\cos^2 x$

8. If for the differential equation $y_1=\frac{y}{x}+\phi\left(\frac{x}{y}\right)$, the general solution is $y=\frac{x}{\log|Cx|}$, then $\phi(x/y)$ is given by

(a) $-x^2/y^2$ (b) y^2/x^2 (c) x^2/y^2 (d) $-y^2/x^2$

9. The solution of $(y(1 + x^{-1}) + \sin y)dx + (x + \log x + x \cos y)dy = 0$ is

(a) $(1 + y^{-1} \sin y) + x^{-1} \log x = C$

(b) $(y + \sin y) + xy \log x = C$

(c) $xy + y \log x + x \sin y = C$

(d) None of these

10. The solution of $y=2x\left(\frac{dy}{dx}\right)+x^2\left(\frac{dy}{dx}\right)^4$ is

(a) $y = 2c^{1/2} x^{1/4} + c$ (b) $y=2\sqrt{c}x^2+c^2$

(c) $y=2\sqrt{c}(x+1)$ (d) $y=2\sqrt{cx}+c^2$

11. The solution of $y_2 - 7y_1 + 12y = 0$ is

(a) $y = C_1e^{3x} + C_2e^{4x}$ (b) $y = C_1xe^{3x} + C_2e^{4x}$

(c) $y = C_1e^{3x} + C_2xe^{4x}$ (d) None of these

12. Solution of the differential equation

$$\frac{x+\frac{x^3}{3!}+\frac{x^5}{5!}+........}{1+\frac{x^2}{2!}+\frac{x^4}{4!}+......}=\frac{dx-dy}{dx+dy}$$ is

(a) $2y\, e^{2x} = C\cdot e^{2x} + 1$ (b) $2y\, e^{2x} = C\cdot e^{2x} - 1$

(c) $y\, e^{2x} = C\cdot e^{2x} + 2$ (d) $2x\, e^{2y} = C\cdot e^x - 1$

13. The solution curves of the differential equation $(xdx+ydy)\sqrt{x^2+y^2}=(xdy-ydx)\left(\sqrt{1-x^2-y^2}\right)$ are

(a) circles of radius 1, passing through the origin

(b) circles of radius 1/2, passing through the origin

(c) circles not passing through origin

(d) solution curve is not a circle

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14. The value of $\frac{d^3x}{dy^3}$ at (0, 1) of the curve

$$y=\frac{x^4+x^2+1}{x^2-x+1} \text{ is}$$

(a) 0 (b) 6 (c) 12 (d) 24

15. The solution of $\frac{dy}{dx}+x(x+y)=x^3(x+y)^3-1$, $y(0)=1$ is given by $(1+x^2)-\frac{1}{(x+y)^2}=f(x)$, where $f(x)=$

(a) 1 (b) e^{-x^2} (c) e^x (d) 0

16. Solution of

$$\frac{y+\sin x\cos^2(xy)}{\cos^2(xy)}dx+\left(\frac{x}{\cos^2(xy)}+\sin y\right)dy=0 \text{ is}$$

(a) $\tan(xy)+\cos x+\cos y=c$

(b) $\tan(xy)-\cos x-\cos y=c$

(c) $\tan(xy)+\cos x-\cos y=c$

(d) $\tan(xy)-\cos x+\cos y=c$

17. The differential equation of all circles in a plane must be

(a) $y_3(1+y_1{}^2)-3y_1\,y_2^2=0$

(b) of order 3 and degree 3

(c) of order 3 and degree 2

(d) $y_3(1-y_1{}^2)-3y_1y_2^2=0$

18. Let $y=f(x)$ be a curve passing through (e, e^e) which satisfy the differential equation $(2ny+xy\log_e x)dx-x\log_e x\,dy=0$, $x>0$, $y>0$. If $g(x)=\lim\limits_{n\to\infty}(0.020)f(x)$, then $\int_{1/e}^{e}g(x)dx$ equal to

(a) e (b) $1/e$ (c) 1 (d) 0

19. The orthogonal trajectories of the system of curves $\left(\frac{dy}{dx}\right)^2=\frac{a}{x}$ are

(a) $9a(y+c)^2=4x^3$ (b) $y+c=\frac{-2}{3\sqrt{a}}x^{3/2}$

(c) $y+c=\frac{2}{3\sqrt{a}}x^{3/2}$ (d) All of these

20. If the solution of $y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=x$, $y(0)=y(1)=1$ is given by $y^2=f(x)$, then

(a) $f(x)$ is monotonically increasing $\forall\ x\in(1,\infty)$

(b) $f(x)=0$ has two real roots

(c) $f(x)$ is even

(d) $f(x)$ has 3 real roots

21. The coordinates of a point $P(x, y)$ are functions of time t and satisfy the relations $\frac{dx}{dt}+\frac{dy}{dt}=t$ and $\frac{dx}{dt}-2\frac{dy}{dt}=t^2$ at any instant of time t. The locus of point $P(x, y)$ is a curve given by (assume $x(0)=y(0)=0$)

(a) $(x+y)^3=(x-2y)^2$

(b) $x=\frac{t^2}{3}-\frac{t^3}{9},\ y=\frac{t^2}{6}-\frac{t^2}{9}$

(c) $9(x+y)^3=8(x-2y)^2$ (d) $8(x+y)^3=9(x-2y)^2$

22. The solution of $p^2+(2y\cot x)p=y^2$, where $p=\frac{dy}{dx}$ is

(a) $y^2(1+\cos x)=c$ (b) $y(1-\cos 2x)=c$

(c) $x=2\sin^{-1}\sqrt{C/2y}$ (d) $x=2\sin^{-1}(\sqrt{2y})+C$

23. Solution of the differential equation :
$(3\tan x+4\cot y-7)\sin^2y\,dx-(4\tan x+7\cot y-5)\cos^2x\,dy=0$ is

(a) $\frac{3}{2}\cot^2x-7\cot x+\frac{7}{2}\tan^2y-5\tan y+4\cot x\cdot\tan y=c$

(b) $\frac{3}{2}\tan^2x-7\tan x+\frac{7}{2}\cot^2y-5\cot y+4\tan x\cdot\cot y=c$

(c) $3\tan^2y-14\cot 2x\cdot\tan^2y+7\cot^2x-10\tan y\cot^2x+8\cot x\cdot\tan y+2c\cot^2x\tan^2y=0$

(d) $3\cot^2y-14\cot x\cdot\cot^2y+7\cot^2x+10\cot y\tan^2x+8\tan x.\cot y=0$

24. If the solution of the differential equation $y^2(y-xp)=x^4p^2$ is in the form $\frac{1}{y}=\frac{c}{x}+c^k$, then $k=$ (Where $p=\frac{dy}{dx}$ and 'c' is arbitrary constant)

(a) 2 (b) 1 (c) 0 (d) –1

25. Differential equation, having $y=(\sin^{-1}x)^2+A(\cos^{-1}x)+B$, where A and B are arbitrary constants is $(p-x^2)\frac{d^2y}{dx^2}-\frac{xdy}{dx}=q$, then $p+q=$

(a) 1 (b) 2 (c) 3 (d) 4

SOLUTIONS

1. **(d) :** Given $\frac{dp}{dx}=p\left(\text{where } p=\frac{dy}{dx}\right)$

$\Rightarrow$ $\ln p=x+c\Rightarrow p=e^{x+c}\Rightarrow\frac{dy}{dx}=ke^x$

$\Rightarrow \quad y = ke^x + \lambda \qquad \ldots(\text{i})$

$\because$ (i) Satisfies (0, 0) $\therefore \lambda = -k$

Thus, $y = k(e^x - 1)$

2. (a) : The parametric form of the given curve is $x = t, y = t^2$.

The equation of any tangent at t is $2xt = y + t^2 \quad \ldots(\text{i})$

On differentiating (i), we get $2t = y_1\left(=\dfrac{dy}{dx}\right)$.

Putting this value in (i), we have

$$2x\frac{y_1}{2} = y + \left(\frac{y_1}{2}\right)^2 \Rightarrow 4xy_1 = 4y + {y_1}^2$$

Thus the order of this equation is 1

3. (a) : We have, $\dfrac{dy}{dx} - f(x)\cdot y = 0$

$$\Rightarrow \quad \frac{dy}{y} = f(x)dx \quad \Rightarrow \quad \ln y = \int f(x)dx$$

$$\Rightarrow \quad y_1(x) = e^{\int f(x)dx}$$

For required equation,

I.F. $= e^{\int f(x)dx}$

Hence solution is $y \cdot y_1(x) = \int r(x)\cdot y_1(x)dx$

i.e., $y = \dfrac{1}{y_1(x)}\displaystyle\int r(x)\cdot y_1(x)dx$

4. (a) : The general eqn. of all parabolas can be written as $y = ax + b \pm \sqrt{2qx + r}$.

Differentiating this equation 4 times ,resulting in the elimination of parameters a, b, q, r gives option (a) as answer.

5. (a) : We have, $y_1y_3 = 3{y_2}^2$

$$\Rightarrow \quad \frac{y_3}{y_2} = 3\frac{y_2}{y_1} \Rightarrow \ln y_2 = 3\ln y_1 + \ln c$$

$$\Rightarrow \quad y_2 = cy_1^3 \quad \Rightarrow \quad \frac{y_2}{y_1^2} = cy_1 \quad \therefore \quad -\frac{1}{y_1} = cy + c_2$$

$$\Rightarrow \quad \frac{dx}{dy} = -cy - c_2 \quad \Rightarrow \quad x = -\frac{cy^2}{2} - c_2y + c_3$$

$\therefore \quad x = A_1y^2 + A_2y + A_3$

6. (a) : We have, $(x^2 \sin^3 y - y^2 \cos x)dx$
$+ (x^3 \cos y \sin^2 y - 2y \sin x)dy = 0$

$\Rightarrow \quad (x^3 \cos y \sin^2 y - 2y \sin x)dy - (y^2 \cos x - x^2 \sin^3 y)dx = 0$

$$\Rightarrow \left(\frac{x^3}{3}d\sin^3 y - \sin x dy^2\right) + \sin^3 y d\left(\frac{x^3}{3}\right) - y^2 d\sin x = 0$$

$$\Rightarrow \frac{x^3}{3}d\sin^3 y + \sin^3 y d\left(\frac{x^3}{3}\right) - \left(\sin x\, dy^2 + y^2 d\sin x\right) = 0$$

$$\Rightarrow d\left(\frac{x^3}{3}\sin^3 y\right) - d\left(y^2 \sin x\right) = 0$$

$$\Rightarrow \quad \frac{x^3}{3}\sin^3 y - y^2 \sin x = c$$

7. (a) : The given equation can be written as

$$2y\sin x\frac{dy}{dx} + y^2 \cos x = \sin 2x$$

$$\Rightarrow \quad \frac{d}{dx}(y^2 \sin x) = \sin 2x$$

$$\Rightarrow y^2 \sin x = (-1/2)\cos 2x + C \qquad \ldots(\text{i})$$

So $(y(\pi/2))^2 \sin(\pi/2) = (-1/2)\cos(2\pi/2) + C$

$\Rightarrow C = 1/2 \qquad \ldots(\text{ii})$

From (i) and (ii), we have

$y^2 \sin x = (1/2)(1 - \cos 2x) = \sin^2 x \Rightarrow y^2 = \sin x$

8. (d) : Putting $v = y/x$ so that $\dfrac{xdv}{dx} + v = \dfrac{dy}{dx}$, we have

$$\frac{xdv}{dx} + v = v + \phi(1/v) \Rightarrow \frac{dv}{\phi(1/v)} = \frac{dx}{x}$$

$\Rightarrow \quad \log|Cx| = \displaystyle\int \frac{dv}{\phi(1/v)}$ (C being constant of integration.)

But $y = \dfrac{x}{\log|Cx|}$ is the general solution

$$\therefore \quad \frac{x}{y} = \frac{1}{v} = \int \frac{dv}{\phi(1/v)} \Rightarrow \phi(1/v) = -v^2 \Rightarrow \phi(x/y) = -y^2/x^2.$$

9. (c) : The given equation can be written as $y(1 + x^{-1})dx + (x + \log x)dy + \sin y dx + x \cos y dy = 0$

$\Rightarrow \quad d(y(x + \log x)) + d(x \sin y) = 0$

$\Rightarrow \quad y(x + \log x) + x \sin y = C$

10. (d) : Writing $p = \dfrac{dy}{dx}$ and differentiating w.r.t. x, we have

$$p = 2p + 2x\frac{dp}{dx} + 2xp^4 + 4p^3x^2\frac{dp}{dx}$$

$$\Rightarrow \quad p(1 + 2xp^3) + 2x\frac{dp}{dx}(1 + 2p^3x) = 0$$

$$\Rightarrow \quad p + 2x\frac{dp}{dx} = 0 \Rightarrow 2\frac{dp}{p} = -\frac{dx}{x}$$

$\Rightarrow \quad 2\log p + \log x = c\,(\text{constant})$

$$\Rightarrow p^2x = c \text{ or } p = \sqrt{\frac{c}{x}}$$

Substituting this value in the given equation, we get $y = 2\sqrt{cx} + c^2$

11. (a) : The given equation can be written as

$$\left(\frac{d}{dx} - 3\right)\left(\frac{dy}{dx} - 4y\right) = 0 \qquad \ldots(\text{i})$$

If $\dfrac{dy}{dx} - 4y = u$ then (i) reduces to $\dfrac{du}{dx} - 3u = 0$

$$\Rightarrow \frac{du}{u} = 3dx \Rightarrow u = Ce^{3x}$$

Therefore, we have $\frac{dy}{dx} - 4y = Ce^{3x}$ ...(ii)

It is a linear equation whose I.F. is e^{-4x}.

Solution of (ii) is given by, $(ye^{-4x}) = \int Ce^{-x}$

$\Rightarrow \quad ye^{-4x} = -Ce^{-x} + C_2 \Rightarrow y = C_1e^{3x} + C_2e^{4x}$

12. (b) : Applying Componendo and Dividendo, we get

$$\frac{dy}{dx} = \frac{e^{-x}}{e^{x}} = e^{-2x} \Rightarrow 2y = -e^{-2x} + C$$

or $\quad 2y\, e^{2x} = C \cdot e^{2x} - 1.$

13. (b) : Put $x = r\cos\theta$, $y = r\sin\theta$

Then, given equation becomes

$$\frac{dr}{\sqrt{1-r^2}} = d\theta \Rightarrow \sin^{-1} r = \theta + \alpha$$

$\Rightarrow \quad r = \sin(\theta + \alpha)$

$\Rightarrow \quad x^2 + y^2 - x\sin\alpha - y\cos\alpha = 0$

$\therefore$ Radius $= \frac{1}{2}$

14. (c) : We use the standard result $\frac{d^3x}{dy^3} = \frac{3y_2^2 - y_1y_3}{y_1^5}$ where $y_n = \frac{d^ny}{dx^n}$. Notice that on dividing $y = x^2 + x + 1$.

15. (d) : Put $x + y = Y$, then given equation becomes $\frac{dY}{dx} + xY = x^3Y^3$.

$$\Rightarrow \frac{1}{Y^3}\frac{dY}{dx} + \frac{x}{Y^2} = x^3 \qquad \text{...(i)}$$

Putting $z = \frac{1}{Y^2}$ in (i), we get $\frac{dz}{dx} - 2xz = -2x^3$

$\Rightarrow \quad z = 1 + x^2 + ce^{x^2} \Rightarrow \quad \frac{1}{(x+y)^2} = 1 + x^2 + ce^{x^2}$

Putting $x = 0$, $y = 1$ gives $c = 0$

16. (b) : $\frac{y\,dx + x\,dy}{\cos^2(xy)} + \sin x dx + \sin y dy = 0$

$$\Rightarrow \frac{d(xy)}{\cos^2(xy)} + \sin x\, dx + \sin y\, dy = 0$$

$\Rightarrow \quad \tan(xy) - \cos x - \cos y = c$

17. (a) : Equation of circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)

Differentiating (i) two times w.r.t. x, we have

$$f = -\frac{1 + yy_2 + y_1^2}{y_2}$$

Again differentiating, we get

$$\frac{y_2\left[2y_1y_2 + y_1y_2 + yy_3\right] - \left[y_3(1 + y_1^2 + yy_2)\right]}{y_2^2} = 0$$

$\Rightarrow \quad 3y_1\,y_2^2 + yy_2y_3 - y_3 - y_3y_1^2 - yy_2y_3 = 0$

$\Rightarrow \quad 3y_1\,y_2^2 = y_3\left[1 + y_1^2\right]$

18. (d) : We have, $(2ny + xy\log_e x)dx = x\log_e x dy$

$$\Rightarrow \frac{dy}{y} = \left(\frac{2n}{x\log_e x} + 1\right)dx$$

$\Rightarrow \quad \log(y) = 2n\log|\log x| + x + c$ and $c = 0$

$$\text{Now, } g(x) = \lim_{n\to\infty} f(x) = \begin{cases} \infty, & \text{if } x < \frac{1}{e} \\ 0, & \text{if } \frac{1}{e} < x < e \\ \infty, & \text{if } x > e \end{cases}$$

$$\therefore \int_{1/e}^{e} g(x)dx = 0$$

19. (d) : Replacing $\frac{dy}{dx}$ by $-dx/dy$, we get

$$\left(\frac{dy}{dx}\right)^2 = x/a \Rightarrow dy = \pm(x^{1/2}/a^{1/2})dx$$

Integrating we get $y + c = \pm\frac{2}{3a^{1/2}}x^{3/2}$

$\Rightarrow \quad 9a(y + c)^2 = 4x^3$

20. (a) : Given,

$$\frac{d}{dx}\left(y\frac{dy}{dx}\right) = x \Rightarrow y\frac{dy}{dx} = \frac{x^2}{2} + c \quad \Rightarrow \frac{d}{dx}\left(\frac{y^2}{2}\right) = \frac{x^2}{2} + c$$

COMIC CAPSULE

First Date with a Logician

These venn diagram will save us a lot of time. We'll see where you fit in, where I fit in, and if any of it overlaps.

$$\Rightarrow\ y^2=\frac{x^3}{3}+\alpha x+\beta \text{ given } y(0)=1,\ y(1)=1$$

$$\Rightarrow\ \beta=1,\ \alpha=\frac{-1}{3}$$

$$\therefore\ \ y^2=f(x)=\frac{x^3-x+3}{3}\ \Rightarrow\ f'(x)=\frac{1}{3}(3x^2-1)>0$$

Hence $f(x)$ is monotonically increasing for $x > 1$

Since, $f\left(\frac{1}{\sqrt{3}}\right)f\left(-\frac{1}{\sqrt{3}}\right)>0$

$\therefore$ $f(x)$ has only one real root.

21. (d) : From the given equations, we get

$$x=\frac{t^2}{3}+\frac{t^3}{9}\ \text{ and }\ y=\frac{t^2}{6}-\frac{t^3}{9}$$

Eliminating t gives $8(x + y)^3 = 9(x - 2y)^2$

22. (c) : Given, $p^2 + 2py\cot x - y^2 = 0$

$$\Rightarrow\ p=\frac{-2y\cot x\pm\sqrt{4y^2\cot^2 x+4y^2}}{2}$$

$$\Rightarrow\ \frac{dy}{dx}=(-\cot x\pm \text{cosec}x)y\ \Rightarrow\ \frac{dy}{dx}+y\frac{(\cos x\pm 1)}{\sin x}=0$$

On integrating, we get

$$\log y+2\log\sin\frac{x}{2}=\log c$$

$$\Rightarrow\ y(1-\cos x)=2c \text{ and } y(1+\cos x)=c$$

$$\Rightarrow\ y\left(2\sin^2 x/2\right)=C\ \Rightarrow\ \sin^2 x/2=\frac{C}{2y}$$

$$\Rightarrow\ x=2\sin^{-1}\sqrt{\frac{C}{2y}}$$

23. (b) : Dividing throughout by $\cos^2x\sin^2y$, the given differential equation becomes

$$(3\tan x + 4\cot y - 7)\sec^2x\,dx - (4\tan x + 7\cot y - 5)\times \text{cosec}^2y\,dy = 0$$

$$\Rightarrow\ 3\tan x\sec^2x\,dx - 7\sec^2x\,dx - 7\cot y\,\text{cosec}^2y\,dy + 5\,\text{cosec}^2y\,dy + 4\cot y\sec^2x\,dx - 4\tan x\,\text{cosec}^2y\,dy = 0$$

$$\Rightarrow\ 3\tan x\,d(\tan x) - 7\,d(\tan x) + 7\cot y\,d(\cot y) - 5\,d(\cot y) + 4\,d(\tan x\cot y) = 0$$

On integrating, we get

$$\frac{3}{2}\tan^2 x-7\tan x+\frac{7}{2}\cot^2 y-5\cot y+4\tan x\cot y=C.$$

24. (a) : Put $x=\frac{1}{X}$ and $y=\frac{1}{Y}$

$$dx=-\frac{1}{X^2}dX \text{ and } dy=-\frac{1}{Y^2}dY$$

$$\Rightarrow\ p=\frac{dy}{dx}=\frac{X^2}{Y^2}\frac{dY}{dX}=\frac{X^2}{Y^2}P$$

The given equation becomes

$$\frac{1}{Y^2}\left(\frac{1}{Y}-\frac{1}{X}\cdot\frac{X^2}{Y^2}P\right)=\frac{1}{X^4}\cdot\frac{X^4}{Y^4}P^2$$

$\Rightarrow$ $Y - XP = P^2$ or $Y = PX + P^2$, which is the Clairaut's form

$\therefore$ The solution is $Y = cX + c^2$ or $\frac{1}{y}=\frac{c}{x}+c^2$

25. (c) : $\frac{dy}{dx}=\frac{2\sin^{-1}x}{\sqrt{1-x^2}}-\frac{A}{\sqrt{1-x^2}}$

$$\Rightarrow\ (1-x^2)(y_1)^2=4y-4B+A^2-2\pi A$$

$$\Rightarrow\ 2(1-x^2)y_1y_2-2x(y_1)^2=4y_1$$

$$\Rightarrow\ (1-x^2)y_2-xy_1=2$$

◆

SOLUTION SENDERS

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Govind Kumar Singh	(Uttar Pradesh)
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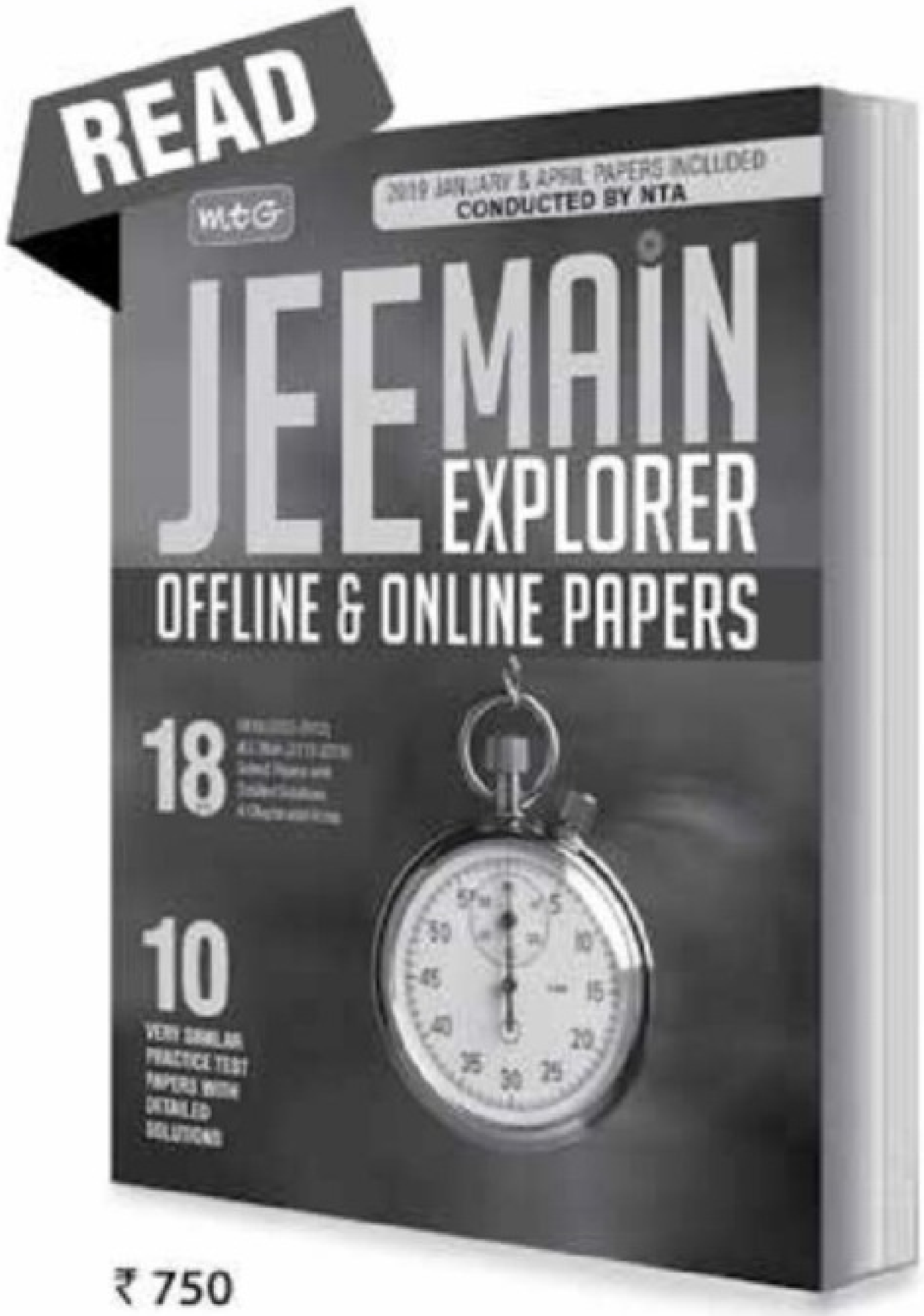
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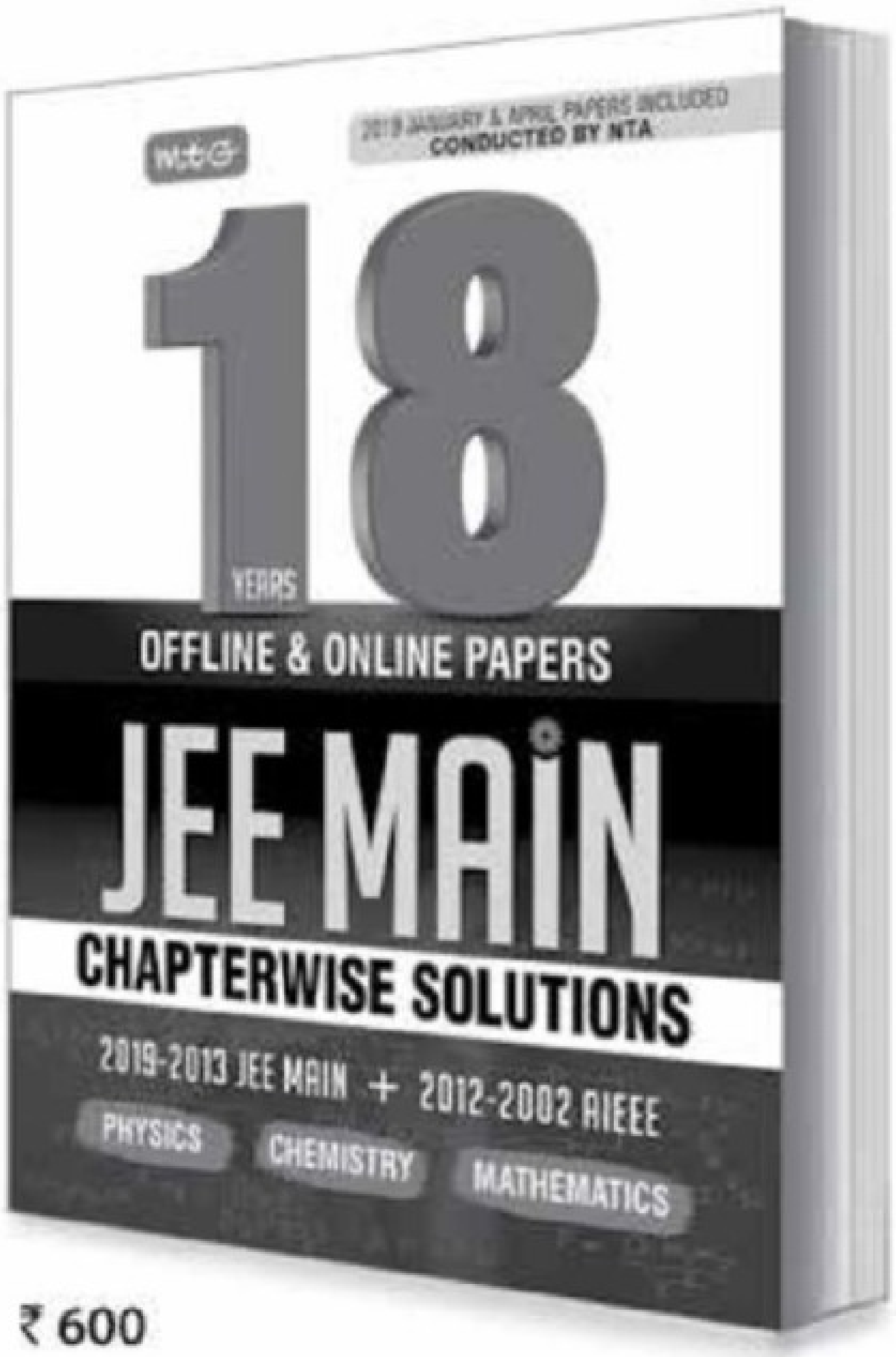
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MOCK TEST PAPER 2020
JEE MAIN

with
Numerical Value
Type Questions

Exam Date : Between 6th to 11th January

Time : 1 hr 15 min. **Series - 6**

The entire syllabus of Mathematics of JEE-MAIN is being divided into eight units, on each unit there will be a Mock Test Paper (MTP) which will be published in the subsequent issues. The syllabus for module break-up is given below:

	Topic	Syllabus in Detail
UNIT NO. 6	Differential Calculus	Derivative of the sum, difference, product and quotient of two functions, chain rule, derivatives of polynomial, rational, trigonometric, inverse trigonometric, exponential and logarithmic functions. Derivative of implicit functions. Derivative up to order two.
	Trigonometry	Inverse trigonometrical functions and their properties. Height and distance.
	Vector Algebra	Vectors and scalars, addition of vectors, components of a vector in two dimensions and three dimensional space, scalar and vector products, scalar and vector triple product.
	Matrices and Determinants	Matrices as a rectangular array of real numbers, equality of matrices, addition, multiplication by a scalar and product of matrices, transpose of a matrix, determinant of square matrix of order up to three, properties of determinants, area of triangles using determinants , inverse of a square matrix of order up to three, properties of these matrix operations, diagonal, symmetric and skew-symmetric matrices and their properties, solutions of simultaneous linear equations in two or three variables.

1. Let $f(x) = \tan 2x \tan 3x \tan 5x$ then $f'(\pi) =$

(a) 10 (b) -10

(c) 0 (d) None of these

2. Let $t = \dfrac{2\sqrt{2}-(1+\sqrt{3})}{\sqrt{3}-1}$, $f(x) = \dfrac{2x}{1-x^2}$ and $g(x) = \dfrac{3x-x^3}{1-3x^2}$, then $\dfrac{d}{dt}(f(g(t))) =$

(a) 1 (b) 2

(c) 0 (d) None of these

3. If $xy = e - e^y$, then $\left.\dfrac{d^2y}{dx^2}\right|_{x=0} =$

(a) $\dfrac{1}{e}$ (b) $\dfrac{1}{e^3}$

(c) $\dfrac{1}{e^2}$ (d) None of these

4. If $x = a\cos^3\theta$, $y = a\sin^3\theta$, then $\sqrt{1+\left(\dfrac{dy}{dx}\right)^2} =$

(a) $\tan^2\theta$ (b) $|\sec\theta|$

(c) $\sec^2\theta$ (d) None of these

5. The derivative of the function

$$f(x) = \cos^{-1}\left\{\frac{1}{\sqrt{13}}(2\cos x - 3\sin x)\right\}$$

$$+\sin^{-1}\left\{\frac{1}{\sqrt{13}}(2\sin x + 3\cos x)\right\}$$ with respect to $\sqrt{1+x^2}$ is

(a) $2x$ (b) $2\sqrt{1+x^2}$

(c) $\dfrac{2}{x}\sqrt{1+x^2}$ (d) $\dfrac{2x}{\sqrt{1+x^2}}$

6. If $D_k = \begin{vmatrix} 1 & 2k & 2k-1 \\ n & n^2+n+1 & n^2 \\ n & n^2+n & n^2+n+1 \end{vmatrix}$ and $\sum_{k=1}^{n} D_k = 72$, then value of n equals

(a) 4 (b) 6 (c) 7 (d) 8

7. For the set of linear equations

$\lambda x - 3y + z = 0$
$x + \lambda y + 3z = 1$
$3x + y + 5z = 2$

the values of λ, for which the equations does not have a unique solution are

By : Sankar Ghosh, S.G.M.C, Mob. 09831244397.

(a) $-1, \frac{11}{5}$ (b) $-1, -\frac{11}{5}$
(c) $-\frac{11}{5}, 1$ (d) $1, \frac{11}{5}$

8. From the matrix equation $AB = AC$, we say $B = C$, provided
(a) A is singular (b) A is square matrix
(c) A is skew symmetric (d) A is non-singular

9. If $f(x) = \begin{vmatrix} 3 & 3x & 3x^2+2a^2 \\ 3x & 3x^2+2a^2 & 3x^3+6a^2x \\ 3x^2+2a^2 & 3x^3+6a^2x & 3x^4+12a^2x^2+2a^4 \end{vmatrix}$, then
(a) $f'(x) = 1$
(b) $y = f(x)$ is a straight line parallel to y-axis
(c) $\int_0^2 f(x)\,dx = 8a^6$ (d) None of these

10. If a square matrix $A = [a_{ij}]$, $a_{ij} = i^2 - j^2$ is of even order then
(a) A is a skew-symmetric and $|A| = 0$
(b) A is symmetric and $|A|$ is a perfect square
(c) A is symmetric and $|A| = 0$
(d) None of these

11. If $\alpha = \sin^{-1}\frac{\sqrt{3}}{2} + \sin^{-1}\frac{1}{3}$ and $\beta = \cos^{-1}\frac{\sqrt{3}}{2} + \cos^{-1}\frac{1}{3}$, then
(a) $\alpha > \beta$ (b) $\alpha = \beta$
(c) $\alpha < \beta$ (d) $\alpha + \beta = 2\pi$

12. The set of the values of x satisfying $2\cos^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$ is
(a) $[0, 1]$ (b) $\left[\frac{1}{\sqrt{2}}, 1\right]$
(c) $\left[0, \frac{1}{\sqrt{2}}\right]$ (d) $[-1, 1]$

13. If $[\cot^{-1}x] + [\cos^{-1}x] = 0$ where 'x' is non-negative real number and $[\cdot]$ denotes the greatest integer function, then complete set of x is
(a) $(\cos 1, 1]$ (b) $(\cos 1, \cot 1)$
(c) $(\cot 1, 1]$ (d) None of these

14. Two flagstaffs stand on a horizontal plane. A and B are two points on the line joining their feet and between them. The angles of elevation of the tops of the flagstaffs as seen from A are 30° and 60° and as seen from B are 60° and 45°. If $AB = 30$ m, then the distance between the flagstaffs in metres is
(a) $30+15\sqrt{3}$ (b) $45+15\sqrt{3}$
(c) $60-15\sqrt{3}$ (d) $60+15\sqrt{3}$

15. The value of

$$\sin^{-1}\left[\cot\left(\sin^{-1}\sqrt{\left(\frac{2-\sqrt{3}}{4}\right)} + \cos^{-1}\frac{\sqrt{12}}{4} + \sec^{-1}\sqrt{2}\right)\right] \text{ is}$$

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

16. A vector of magnitude $\sqrt{2}$ coplanar with $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$ and perpendicular to $\hat{i}+\hat{j}+\hat{k}$ is
(a) $-\hat{j}+\hat{k}$ (b) $\hat{i}-\hat{j}$
(c) $\hat{i}-\hat{j}$ (d) $\hat{i}-2\hat{j}+\hat{k}$

17. Let $\vec{a} = 2\hat{i}-\hat{j}+\hat{k}, \vec{b} = \hat{i}+2\hat{j}-\hat{k}$ and $\vec{c} = \hat{i}+\hat{j}-2\hat{k}$ be three vectors. A vector in the plane of $\vec{b}$ and $\vec{c}$ whose projection on $\vec{a}$ is of magnitude $\sqrt{\frac{2}{3}}$ is
(a) $2\hat{i}-3\hat{j}-3\hat{k}$ (b) $2\hat{i}+3\hat{j}+3\hat{k}$
(c) $-2\hat{i}-\hat{j}+5\hat{k}$ (d) $2\hat{i}+\hat{j}+5\hat{k}$

18. If $|\vec{a}| = 4, |\vec{b}| = 2$ and the angle between the vectors $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then $|\vec{a}\times\vec{b}|^2 =$
(a) 48 (b) 16
(c) 64 (d) None of these

19. If $\vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{c}\times\vec{a} \neq \vec{0}$, then $\vec{a}+\vec{b}+\vec{c} =$
(a) $-\vec{b}$ (b) $\vec{0}$
(c) $2\vec{a}$ (d) None of these

20. If the unit vectors $\vec{a}$ and $\vec{b}$ are inclined at an angle 2θ, $0 \le \theta \le \pi$ and $|\vec{a}-\vec{b}|$ is less than 1, then θ lies in the interval
(a) $\left[0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, \pi\right]$ (b) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, \pi\right)$
(c) $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$ (d) $\left[\frac{\pi}{2}, \frac{5\pi}{6}\right]$

NUMERICAL VALUE TYPE

21. If $A = \begin{pmatrix} i & -i \\ -i & i \end{pmatrix}$ (where $i = \sqrt{-1}$) and $B = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ then $A^4 = kB$. The value of k is

22. If $f(x)=\begin{vmatrix}1+\sin^2 x & \cos^2 x & 4\sin 2x\\ \sin^2 x & 1+\cos^2 x & 4\sin 2x\\ \sin^2 x & \cos^2 x & 1+4\sin 2x\end{vmatrix}$, then maximum value of $f(x)$ is

23. Suppose $a, b, c \in R$ and $abc = 1$. If $A=\begin{bmatrix}2a & b & c\\ b & 2c & a\\ c & a & 2b\end{bmatrix}$ is such that $A'A=64^{\frac{1}{3}}I$ and $|A| > 0$, then the value of $a^3 + b^3 + c^3$ is

24. If $y = \sin(4\sin^{-1}x)$ satisfies the relation $(1 - x^2)y_2 - xy_1 + 4Ay = 0$, then A is equal to

25. If $y=\left(1+\frac{1}{x}\right)^x$ then $\dfrac{2\sqrt{y_2(2)+\frac{1}{8}}}{\log\frac{3}{2}-\frac{1}{3}}=$

SOLUTIONS

1. (c) : We know that

$$\tan 5x=\tan(2x+3x)=\frac{\tan 2x+\tan 3x}{1-\tan 2x\tan 3x}$$

$\Rightarrow$ $\tan 2x \tan 3x \tan 5x = \tan 5x - \tan 3x - \tan 2x$

$\therefore$ $f(x) = \tan 2x \tan 3x \tan 5x = \tan 5x - \tan 3x - \tan 2x$

$\Rightarrow$ $f'(x) = 5\sec^2 5x - 3\sec^2 3x - 2\sec^2 2x$

$\Rightarrow$ $f'(\pi) = 5(1) - 3(1) - 2(1)$ $[\because\ \cos n\pi = (-1)^n] = 0$

2. (c) : We have,

$$t=\frac{2\sqrt{2}-(1+\sqrt{3})}{\sqrt{3}-1}=\frac{2\sqrt{2}(\sqrt{3}+1)-(\sqrt{3}+1)^2}{3-1}$$

$$=\frac{2\sqrt{6}+2\sqrt{2}-4-2\sqrt{3}}{2}=\sqrt{6}+\sqrt{2}-2-\sqrt{3}$$

$$=\sqrt{6}-\sqrt{4}-\sqrt{3}+\sqrt{2}=\tan 7\frac{1}{2}^\circ$$

Now, $f(x)=\dfrac{2x}{1-x^2}\Rightarrow f(t)=\tan 15^\circ=(2-\sqrt{3})$

and $g(x)=\dfrac{3x-x^3}{1-3x^2}\Rightarrow g(t)=\tan\left(22\frac{1}{2}\right)^\circ=\sqrt{2}-1$

$$\therefore\quad f(g(t))=f\left(\tan\left(22\frac{1}{2}\right)^\circ\right)$$

$$=\frac{2\tan\left(22\frac{1}{2}\right)^\circ}{1-\tan\left(22\frac{1}{2}^\circ\right)^2}=\tan 2\left(22\frac{1}{2}^\circ\right)=\tan 45^\circ=1$$

$$\therefore\quad \frac{d}{dt}(f(g(t)))=\frac{d}{dt}(1)=0\,.$$

3. (c) : We have, $xy = e - e^y$ ...(i)

$\Rightarrow\quad y+x\dfrac{dy}{dx}=-e^y\dfrac{dy}{dx}\Rightarrow e^y\dfrac{dy}{dx}+x\dfrac{dy}{dx}+y=0$ ...(ii)

$\therefore\quad \left.\dfrac{dy}{dx}\right|_{x=0}=-\dfrac{1}{e}$ [As $x=0\Rightarrow y=1$ by(i)]

Again differentiating (ii) w.r.t. x we get,

$e^y\dfrac{d^2y}{dx^2}+e^y\left(\dfrac{dy}{dx}\right)^2+x\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}=0$ ...(iii)

Putting $x=0$, $y=1$ and $\dfrac{dy}{dx}=-\dfrac{1}{e}$ in (iii) we get

$$\frac{d^2y}{dx^2}=\frac{1}{e^2}$$

4. (b) : We have, $x = a\cos^3\theta$, $y = a\sin^3\theta$

$$\therefore\quad \frac{dx}{d\theta}=-3a\sin\theta\cos^2\theta \text{ and } \frac{dy}{d\theta}=3a\sin^2\theta\cos\theta$$

$$\therefore\quad \frac{dy}{dx}=-\tan\theta\Rightarrow\left(\frac{dy}{dx}\right)^2=\tan^2\theta$$

$$\Rightarrow\quad 1+\left(\frac{dy}{dx}\right)^2=1+\tan^2\theta=\sec^2\theta=|\sec\theta|^2$$

$$\therefore\quad \sqrt{1+\left(\frac{dy}{dx}\right)^2}=|\sec\theta|$$

5. (c) : $f(x)=\cos^{-1}\left\{\dfrac{1}{\sqrt{13}}(2\cos x-3\sin x)\right\}$
$\qquad+\sin^{-1}\left\{\dfrac{1}{\sqrt{13}}(2\sin x+3\cos x)\right\}$

Let $\cos\alpha=\dfrac{2}{\sqrt{13}}$ and $\sin\alpha=\dfrac{3}{\sqrt{13}}$

$\therefore$ $f(x) = \cos^{-1}(\cos\alpha\cos x - \sin\alpha\sin x)$
$\qquad + \sin^{-1}(\cos\alpha\sin x + \sin\alpha\cos x)$

$= \cos^{-1}(\cos(x + \alpha)) + \sin^{-1}(\sin(x + \alpha)) = 2x + 2\alpha$

$\therefore\quad \dfrac{d}{dx}(f(x))=2$ ...(i)

Let $g(x)=\sqrt{1+x^2}$

$\Rightarrow\dfrac{d}{dx}(g(x))=\dfrac{1}{2\sqrt{1+x^2}}\times 2x=\dfrac{x}{\sqrt{1+x^2}}$ ...(ii)

From (i) and (ii), the required derivative $=\dfrac{2}{x}\sqrt{1+x^2}$.

6. (d) : Given,

$$D_k=\begin{vmatrix}1 & 2k & 2k-1\\ n & n^2+n+1 & n^2\\ n & n^2+n & n^2+n+1\end{vmatrix}\text{ and }\sum_{k=1}^{n}D_k=72$$

$$\therefore \sum_{k=1}^{n} D_k = \begin{vmatrix} \sum_{k=1}^{n} 1 & \sum_{k=1}^{n} 2k & \sum_{k=1}^{n}(2k-1) \\ n & n^2+n+1 & n^2 \\ n & n^2+n & n^2+n+1 \end{vmatrix}$$

$$= \begin{vmatrix} n & n(n+1) & n^2 \\ n & n^2+n+1 & n^2 \\ n & n^2+n & n^2+n+1 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\begin{vmatrix} n & n(n+1) & n^2 \\ 0 & 1 & 0 \\ 0 & 0 & n+1 \end{vmatrix} = n(n+1)$$

Hence, $n(n + 1) = 72 = 8(8 + 1) \Rightarrow n = 8$

7. (a) : Given that, the system has no unique solution. Therefore either have no solution or infinite solution.

$\therefore$ $|A| = 0$ for both cases.

$$\text{Now, } \begin{vmatrix} \lambda & -3 & 1 \\ 1 & \lambda & 3 \\ 3 & 1 & 5 \end{vmatrix} = 0 \Rightarrow 5\lambda^2 - 6\lambda - 11 = 0$$

$$\Rightarrow (5\lambda - 11)(\lambda + 1) = 0 \Rightarrow \lambda = \frac{11}{5}, \lambda = -1$$

8. (d) : Let $|A| \neq 0$ $\therefore$ A^{-1} exist

Given that $AB = AC$ $\therefore$ $A^{-1}(AB) = A^{-1}(AC) \Rightarrow B = C$

(By associative property)

$\Rightarrow$ A is non-singular.

9. (c) : We have,

$$f(x) = \begin{vmatrix} 3 & 3x & 3x^2+2a^2 \\ 3x & 3x^2+2a^2 & 3x^3+6a^2x \\ 3x^2+2a^2 & 3x^3+6a^2x & 3x^4+12a^2x^2+2a^4 \end{vmatrix}$$

Applying $C_3 \to C_3 - C_2x$ and $C_2 \to C_2 - C_1x$

$$\begin{vmatrix} 3 & 0 & 2a^2 \\ 3x & 2a^2 & 4a^2x \\ 3x^2+2a^2 & 4a^2x & 6a^2x^2+2a^4 \end{vmatrix}$$

Applying $R_3 \to R_3 - R_2x$ and $R_2 \to R_2 - R_1x$

$$\begin{vmatrix} 3 & 0 & 2a^2 \\ 0 & 2a^2 & 2a^2x \\ 2a^2 & 2a^2x & 2a^2x^2+2a^4 \end{vmatrix} = a^4 \begin{vmatrix} 3 & 0 & 2a^2 \\ 0 & 2 & 2x \\ 2 & 2x & 2x^2+2a^2 \end{vmatrix}$$

$= a^4[3(4x^2 + 4a^2 - 4x^2) + 2a^2(-4)] = a^4(12a^2 - 8a^2) = 4a^6$

$$\therefore f(x) = 4a^6 \Rightarrow f'(x) = 0 \text{ and } \int_0^2 4a^6 dx = 4a^6[x]_0^2 = 8a^6$$

10. (d) : We have,

$$A = [a_{ij}], a_{ij} = i^2 - j^2 \therefore A = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix} \text{ and } |A| = 9$$

11. (c) : We have,

$$\alpha = \sin^{-1}\frac{\sqrt{3}}{2} + \sin^{-1}\frac{1}{3} \text{ and } \beta = \cos^{-1}\frac{\sqrt{3}}{2} + \cos^{-1}\frac{1}{3}$$

$$\therefore \alpha + \beta = \sin^{-1}\frac{\sqrt{3}}{2} + \sin^{-1}\frac{1}{3} + \cos^{-1}\frac{\sqrt{3}}{2} + \cos^{-1}\frac{1}{3}$$

$$= \left(\sin^{-1}\frac{\sqrt{3}}{2} + \cos^{-1}\frac{\sqrt{3}}{2}\right) + \left(\sin^{-1}\frac{1}{3} + \cos^{-1}\frac{1}{3}\right)$$

$$= \frac{\pi}{2} + \frac{\pi}{2} \quad \left[\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$$

$$= \pi$$

$$\text{Also, } \alpha = \sin^{-1}\frac{\sqrt{3}}{2} + \sin^{-1}\frac{1}{3} = \frac{\pi}{3} + \sin^{-1}\frac{1}{3} < \frac{\pi}{3} + \sin^{-1}\frac{1}{2}$$

$$\left[\text{Since, } \sin\theta \text{ is increasing in } \left(0, \frac{\pi}{2}\right)\right]$$

$$< \frac{\pi}{3} + \frac{\pi}{6} = \frac{\pi}{2}. \text{ It follows that } \beta > \frac{\pi}{2} > \alpha.$$

12. (b) : We have, $2\cos^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

Let $\cos^{-1}x = \theta$, LHS $= 2\theta$ when $0 \le \theta \le \pi$ ...(i)

again, RHS $= \sin^{-1}(2\sin\theta\cos\theta) = \sin^{-1}(\sin 2\theta)$

$$\Rightarrow -\frac{\pi}{4} \le \theta \le \frac{\pi}{4} \quad \text{...(ii)}$$

From (i) and (ii), we have

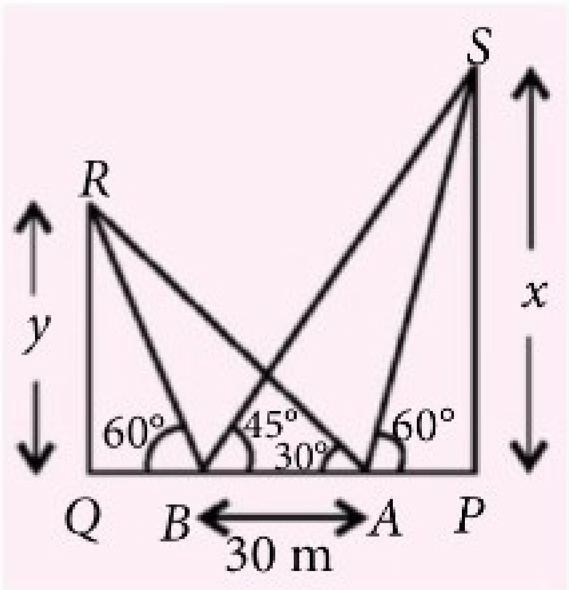
$$0 \le \theta \le \frac{\pi}{4} \Rightarrow 0 \le \cos^{-1}x \le \frac{\pi}{4} \Rightarrow \frac{1}{\sqrt{2}} \le x \le 1 \Rightarrow x \in \left[\frac{1}{\sqrt{2}}, 1\right]$$

13. (c) : We have, $[\cot^{-1}x] + [\cos^{-1}x] = 0$

$\Rightarrow$ $[\cot^{-1}x] = 0$ and $[\cos^{-1}x] = 0$

$\Rightarrow$ $x \in (\cot 1, \infty)$ and $x \in [\cos 1, 1] \Rightarrow x \in (\cot 1, 1]$

14. (d) : Let x and y be the height of the flagstaffs PS and QR respectively.



$$\text{Then } AP = x\cot 60^\circ = \frac{x}{\sqrt{3}},$$

$$AQ = y\cot 30^\circ = y\sqrt{3}$$

$$\text{and } BQ = y\cot 60^\circ = \frac{y}{\sqrt{3}}$$

$$AB = BP - AP = x - \frac{x}{\sqrt{3}}$$

$$\Rightarrow 30 = \frac{(\sqrt{3}-1)}{\sqrt{3}}x \Rightarrow x = 15(3+\sqrt{3})$$

Again, $AB = AQ - BQ = \sqrt{3}y - \dfrac{y}{\sqrt{3}}$

$$\Rightarrow \quad 30 = \frac{(3-1)}{\sqrt{3}}y \Rightarrow y = 15\sqrt{3}$$

So, $PQ = BP + BQ = x + \dfrac{y}{\sqrt{3}} = 15(3+\sqrt{3}) + 15$

$$= 60 + 15\sqrt{3} \text{ metres}$$

15. (a) : We have,

$$\sin^{-1}\left[\cot\left(\sin^{-1}\sqrt{\left(\frac{2-\sqrt{3}}{4}\right)} + \cos^{-1}\frac{\sqrt{12}}{4} + \sec^{-1}\sqrt{2}\right)\right]$$

$$= \sin^{-1}\left[\cot\left(\sin^{-1}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right) + \cos^{-1}\frac{\sqrt{3}}{2} + \sec^{-1}\sqrt{2}\right)\right]$$

$= \sin^{-1}[\cot(15° + 30° + 45°)]$

$= \sin^{-1}[\cot 90°] = \sin^{-1}(0) = 0$

16. (a) : A vector coplanar with $\hat{i} + \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$ is given by $\hat{i} + \hat{j} + 2\hat{k} + \lambda(\hat{i} + 2\hat{j} + \hat{k})$

$= (1+\lambda)\hat{i} + (1+2\lambda)\hat{j} + (2+\lambda)\hat{k}$

Now, $(1+\lambda)\hat{i} + (1+2\lambda)\hat{j} + (2+\lambda)\hat{k}$ is perpendicular to $\hat{i} + \hat{j} + \hat{k}$

$\therefore \quad (1 + \lambda)\cdot 1 + (1 + 2\lambda)\cdot 1 + (2 + \lambda)\cdot 1 = 0 \Rightarrow \lambda = -1$

Hence, the required vector is $-\hat{j} + \hat{k}$ and its magnitude is $|-\hat{j} + \hat{k}| = \sqrt{2}$

17. (c) : Let $\vec{R}$ be a vector in the plane of $\vec{b}$ and $\vec{c}$.

Then, $\vec{R} = \vec{b}\ + \lambda\ \vec{c} = \hat{i} + 2\hat{j} - \hat{k} + \lambda(\hat{i} + \hat{j} - 2\hat{k})$

$= (1+\lambda)\hat{i} + (2+\lambda)\hat{j} - (1+2\lambda)\hat{k}$

Now, its projection on $\vec{a} = \dfrac{\vec{a}\cdot\vec{R}}{|\vec{a}|}$

$$= \frac{(2\hat{i} - \hat{j} + \hat{k})\cdot\{(1+\lambda)\hat{i} + (2+\lambda)\hat{j} - (1+2\lambda)\hat{k}\}}{\sqrt{4+1+1}}$$

$$= \frac{2(1+\lambda) - 1(2+\lambda) - 1(1+2\lambda)}{\sqrt{6}} = \frac{-(1+\lambda)}{\sqrt{6}}$$

$$\therefore \quad \frac{-(1+\lambda)}{\sqrt{6}} = \pm\sqrt{\frac{2}{3}} \Rightarrow -(1+\lambda) = \pm 2 \Rightarrow \lambda = 1, -3$$

$$\Rightarrow \quad \vec{R} = 2\hat{i} + 3\hat{j} - 3\hat{k} \text{ and } -2\hat{i} - \hat{j} + 5\hat{k}$$

18. (b) : Here, $|\vec{a}| = 4,\ |\vec{b}| = 2$

Now, $|\vec{a}\times\vec{b}| = |\vec{a}|\ |\vec{b}|\sin\theta$

$[\because |n| = 1 \text{ and } 0 \le \theta \le \pi \ \ \therefore \sin\theta > 0]$

$$\Rightarrow \quad |\vec{a}\times\vec{b}|^2 = 4^2 \times 2^2 \times \sin^2\frac{\pi}{6} = 16\times 4\times\frac{1}{4} = 16$$

19. (b) : We have, $\vec{a}\times\vec{b} = \vec{b}\times\vec{c} \Rightarrow \vec{a}\times\vec{b} = -\vec{c}\times\vec{b}$

$\Rightarrow \quad \vec{a}\times\vec{b} + \vec{c}\times\vec{b} = \vec{0} \ \Rightarrow \ (\vec{a}+\vec{c})\times\vec{b} = \vec{0}$

$\Rightarrow \quad (\vec{a}+\vec{b}+\vec{c})\times\vec{b} = \vec{0} \quad [\because \ \vec{b}\times\vec{b} = \vec{0}]$

Similarly, $\vec{b}\times\vec{c} = \vec{c}\times\vec{a} \Rightarrow (\vec{a}+\vec{b}+\vec{c})\times\vec{c} = \vec{0}$

$\because \quad \vec{b}, \vec{c}$ are non-collinear $\therefore \vec{a}+\vec{b}+\vec{c} = \vec{0}$

20. (a) : Given that $\vec{a}$ and $\vec{b}$ are unit vectors and angle between them is 2θ and $|\vec{a}-\vec{b}| < 1$

$\therefore \ |\vec{a}-\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\cdot\vec{b} = 2(1 - \cos 2\theta) = 4\sin^2\theta$

$$\Rightarrow \quad |\vec{a}-\vec{b}| = 2|\sin\theta| \Rightarrow |\sin\theta| < \frac{1}{2}$$

$$\therefore \quad \theta \in \left[0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, \pi\right]$$

21. (8) : We have, $A = \begin{pmatrix} i & -i \\ -i & i \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$

Now, $A^2 = \begin{pmatrix} i & -i \\ -i & i \end{pmatrix}\begin{pmatrix} i & -i \\ -i & i \end{pmatrix} = \begin{pmatrix} i^2+i^2 & -i^2-i^2 \\ -i^2-i^2 & i^2+i^2 \end{pmatrix}$

$$= \begin{pmatrix} 2i^2 & -2i^2 \\ -2i^2 & 2i^2 \end{pmatrix} = \begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix} = 2\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\therefore \quad A^4 = 2\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}\times 2\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$= 4\begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix} = 8\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \quad \Rightarrow \quad A^4 = 8B$$

Hence, $k = 8$.

22. (6) : Given,

$$f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{vmatrix}$$

Applying $R_1 \to R_1 - R_3$ and $R_2 \to R_2 - R_3$, we get

$$f(x) = \begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{vmatrix}$$

$= 1(1 + 4\sin 2x + \cos^2 x) - 1(-\sin^2 x)$

$= 1 + 4\sin 2x + \cos^2 x + \sin^2 x$

$= 1 + 4\sin 2x + 1 = 2 + 4\sin 2x$

Thus, $f(x)$ will be maximum when $\sin 2x = 1$.

Therefore the maximum value of $f(x)$ is 6.

23. (4) : Given that $A'A = 64^{1/3}I$

$\Rightarrow \quad |A'A| = |64^{1/3}I| = 4|I| = 4$

$\Rightarrow \quad |A|^2 = 4 \Rightarrow |A| = 2 \qquad \ldots(i)$

Now, $|A| = \begin{vmatrix} 2a & b & c \\ b & 2c & a \\ c & a & 2b \end{vmatrix}$

$$= 2a(4bc - a^2) - b(2b^2 - ac) + c(ab - 2c^2)$$
$$= 8abc - 2a^3 - 2b^3 + abc + abc - 2c^3$$
$$= 10abc - 2(a^3 + b^3 + c^3)$$
$$\therefore \quad 2 = 10abc - 2(a^3 + b^3 + c^3) \qquad \text{[From (i)]}$$
$$\Rightarrow \quad 2(a^3 + b^3 + c^3) = 10 - 2 = 8 \qquad [\because abc = 1]$$
$$\Rightarrow \quad a^3 + b^3 + c^3 = 4$$

24. (4) : We have, $y = \sin(4\sin^{-1}x)$

Differentiating w.r.t. x, we get

$$y_1 = \cos(4\sin^{-1}x)\cdot\frac{4}{\sqrt{1-x^2}}$$
$$\Rightarrow \quad \sqrt{1-x^2}\,y_1 = 4\cos(4\sin^{-1}x)$$
$$\Rightarrow \quad (1 - x^2)y_1^2 = 16\cos^2(4\sin^{-1}x)$$
$$= 16\{(1 - \sin^2(4\sin^{-1}x)\} = 16 - 16y^2$$

Again differentiating w.r.t. x, we get

$$(1 - x^2)2y_1y_2 - 2xy_1^2 = -32yy_1$$
$$\Rightarrow \quad (1 - x^2)y_2 - xy_1 + 16y = 0$$

25. (3) : We have, $y = \left(1+\frac{1}{x}\right)^x$

$$\Rightarrow \quad \log y = x\log\left(\frac{1+x}{x}\right) \qquad \text{...(i)}$$

Differentiating (i) w.r.t. x, we get

$$\frac{1}{y}\cdot\frac{dy}{dx} = x\left[\frac{1}{1+x} - \frac{1}{x}\right] + \log\left(\frac{1+x}{x}\right)\cdot 1$$
$$= \log\left(\frac{1+x}{x}\right) - \frac{1}{1+x}$$
$$\Rightarrow \quad \frac{dy}{dx} = y\left[\log\left(\frac{1+x}{x}\right) - \frac{1}{1+x}\right] \qquad \text{...(ii)}$$
$$\Rightarrow \quad \left[\frac{dy}{dx}\right]_{x=2} = y(2)\left[\log\left(\frac{1+2}{2}\right) - \frac{1}{1+2}\right]$$
$$= \frac{9}{4}\left[\log\frac{3}{2} - \frac{1}{3}\right] \qquad \left[\because \; y(2) = \frac{9}{4}\right]$$

Now, differentiating (ii) w.r.t. x, we get

$$\frac{d^2y}{dx^2} = y\left[\frac{1}{1+x} - \frac{1}{x} + \frac{1}{(1+x)^2}\right] + \left[\log\left(\frac{1+x}{x}\right) - \frac{1}{1+x}\right]\frac{dy}{dx}$$
$$= y\left[\frac{1}{(1+x)^2} - \frac{1}{x(x+1)}\right] + \left[\log\left(\frac{1+x}{x}\right) - \frac{1}{1+x}\right]\frac{dy}{dx}$$
$$\Rightarrow \quad \left[\frac{d^2y}{dx^2}\right]_{x=2} = y(2)\left(\frac{1}{9} - \frac{1}{6}\right) + \left(\log\frac{3}{2} - \frac{1}{3}\right)\left[\frac{dy}{dx}\right]_{x=2}$$
$$= \frac{9}{4}\left(-\frac{1}{18}\right) + \left(\log\frac{3}{2} - \frac{1}{3}\right)\times\frac{9}{4}\left(\log\frac{3}{2} - \frac{1}{3}\right)$$
$$= \frac{9}{4}\left(\log\frac{3}{2} - \frac{1}{3}\right)^2 - \frac{1}{8}$$

Now, $$\left[\frac{d^2y}{dx^2}\right]_{x=2} + \frac{1}{8} = \frac{9}{4}\left(\log\frac{3}{2} - \frac{1}{3}\right)^2 - \frac{1}{8} + \frac{1}{8}$$
$$= \frac{9}{4}\left(\log\frac{3}{2} - \frac{1}{3}\right)^2$$

Hence, $$\frac{2\sqrt{y_2(2) + \frac{1}{8}}}{\log\frac{3}{2} - \frac{1}{3}} = \frac{2\sqrt{\left[\frac{d^2y}{dx^2}\right]_{x=2} + \frac{1}{8}}}{\log\frac{3}{2} - \frac{1}{3}}$$
$$= \frac{2\times\frac{3}{2}\left(\log\frac{3}{2} - \frac{1}{3}\right)}{\left(\log\frac{3}{2} - \frac{1}{3}\right)} = 3$$

◆

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique answer.

			7	3				1				3		5	1	7	9		8	6
4			2	6	1	7	3					8			3					
	7		8		4								7				8			3
6	1				3		8						6				5		4	
	4						7							2			7		5	
		8						2				9				1				
				1	8		2				7			3					6	
	2	5		4			1	6	8	9	2			7		8			9	2
3			6			8				1				6		4	1			
							8			2	4			1						
						1	3					6								
						7		4		5		3		8						
		8	1	2		4		3		7	1	8			9	1		3		
2				8	6		7			6		4				2			1	
	1				7				4					9		4		8		
				3	8		4					6	4	8			2	9	3	
6				7		1		9				5								
9	3			6		5								7			9	6		
				1													8	1	7	
8	7							5				3			5	9			8	
4	9	2					1	7				2	8			7				

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

OLYMPIAD CORNER

1. Prove that for all positive intergers n, $p_n(x) = x^n - 2$ is irreducible in $Q[x]$.
2. If the sum of the proper divisors of n (that is, the divisors of n that are less than n)is n , then n is called a perfect number and the sum of the divisors of n, $\sigma(n)$ is $2n$. then, find all even perfect numbers.
3. Let r be a line in the plane and let ABC be a triangle contained in one of the half-planes determined by r. Let A', B', C', be the points symmetric to A, B, C with respect to r; draw the line through A' parallel to BC, the line through B' parallel to AC and the line through C' parallel to AB. Show that these three lines have a common point.
4. Determine all the possible values of the sum of the digits of the perfect squares.
5. Little sam is a unique child and his math marks show it. On four tests this year his scores out of 100 were all two-digit numbers made up of eight different non-zero digits. Also, the average of these scores is the same as the average if each score is reversed and this average is an integer none of whose digits is equal to any of the digits in the scores. What is sam's average?
6. Prove the inequality
$$\left(\sum_{i=1}^{n} a_i\right)\left(\sum_{i=1}^{n} b_i\right) \geq \left(\sum_{i=1}^{n} (a_i+b_i)\right)\left(\sum_{i=1}^{n} \frac{a_i b_i}{a_i+b_i}\right)$$
for any positive numbers $a_1, \ldots, a_n, b_1, \ldots, b_n$.
7. A circular billiard table is given with a cue ball at the circumference. It is shot at an angle of α to the line from the ball to the centre of the table. For what angles α will the ball come back to this point, assuming the ball keeps going indefinitely?

SOLUTIONS

1. This follows directly from Eisenstein's criterion, but we will take a more basic approach. By Demoivre's theorem, the n roots of $p_n(x)$ (in C) are $2^{\frac{1}{n}} \operatorname{cis}\left(\frac{2k\pi}{n}\right)$, $k = 0, 1, 2, \ldots n - 1$, so that

$$p_n(x) = \left(x - 2^{\frac{1}{n}} \operatorname{cis} 0\right)\left(x - 2^{\frac{1}{n}} \operatorname{cis}\left(\frac{2\pi}{n}\right)\right)\ldots \left(x - 2^{\frac{1}{n}} \operatorname{cis}\left(\frac{2(n-1)\pi}{n}\right)\right)$$

Suppose $p_n(x) = f(x)\, g(x)$ for some $f(x), g(x) \in Q[x]$, with $\deg f(x) = m$ and $n > m > 0$. (Note : We only need to consider $n \geq 2$). Let the roots of $f(x)$, respectively $g(x)$, be $w_1, w_2, \ldots w_m$, respectively $w_{m+1}, w_{m+2}, \ldots w_n$, so $w_1, w_2, \ldots w_n$ is permutation of the roots of $p_n(x)$.

Let $w_i = 2^{1/n} \operatorname{cis} \theta_i$, and let $\theta = \sum_{k=1}^{m} \theta_i$.

Since, $(-1)^m \prod_{k=1}^{m} w_k$ is constant term of $f(x)$, so

$$(-1)^m \prod_{k=1}^{m} w_k = (-1)^m 2^{\frac{m}{n}} \prod_{k=1}^{m} \operatorname{cis} \theta_k = (-1)^m 2^{\frac{m}{n}} \operatorname{cis} \theta \in Q.$$

But $\operatorname{cis} \theta = \cos\theta + i \sin\theta \in Q$ implies that $\sin\theta = 0$ and further, that $\cos\theta = \pm 1$. Therefore, the constant term of $f(x)$ is $\pm 2^{\frac{m}{n}}$. It is easy to show that $2^{\frac{m}{n}} \notin Q$, a contradiction. Hence, $x^n - 2$ is irreducible in $Q[x]$.

2. By the Fundamental Theorem of Arithmetic, we can express n uniquely, up to permutations, is the form $p_1^{a_1} p_2^{a_2} \ldots p_k^{a_k}$, where the p_i are distinct primes and the a_i are positive integers. Then, all divisors of n are of the form $p_1^{a_1} p_2^{a_2} \ldots p_k^{a_k}$, where $0 \leq a_i \leq a_i$.

$$\text{Hence, } \sigma(n) = \prod_{j=1}^{k} \left(1 + p_j + p_j^2 + \ldots + p_j^{a_j}\right)$$

$$= \left(\frac{p_1^{a_1+1} - 1}{p_1 - 1}\right)\left(\frac{p_2^{a_2+1} - 1}{p_2 - 1}\right) \ldots \left(\frac{p_k^{a_k+1} - 1}{p_k - 1}\right).$$

It follows that if $\gcd(m, n) = 1$, then, $\sigma(mn) = \sigma(m)\sigma(n)$. We claim that if $2^n - 1$ is prime, then $2^{n-1}(2^n - 1)$ is a perfect number.

In such a case, we have

$$\sigma(2^{n-1}(2^n-1))=\frac{2^n-1}{2-1}\cdot\frac{(2^n-1)^2-1}{(2^n-1)-1}$$

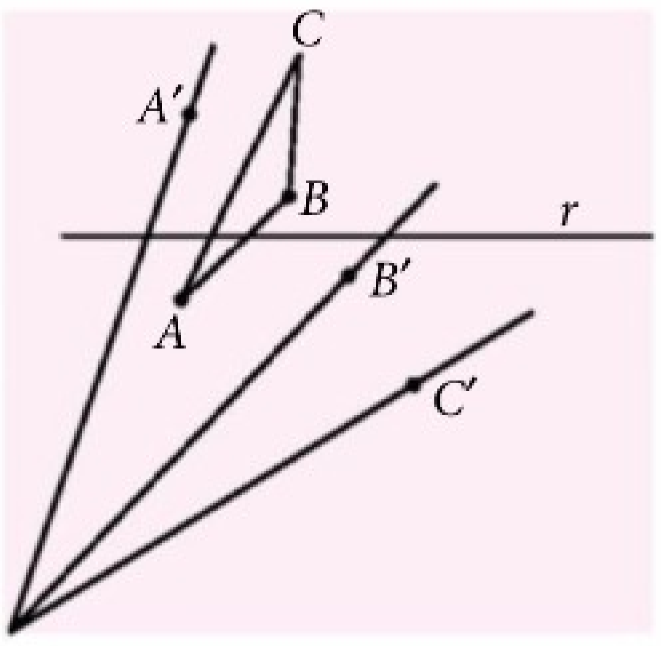
$$=(2^n-1)(2^n-1+1)=2\cdot 2^{n-1}(2^n-1).$$

Now consider an arbitrary even perfect number n. Then, $n = 2^k m$, where $k \geq 1$ and gcd $(2^k, m) = 1$. Also, $2^{k+1}m = \sigma(2^k m) = \sigma(2^k)\,\sigma(m) = (2^{k+1} - 1)\,\sigma(m)$. Now, $2^{k+1} - 1$ is odd, so $(2^{k+1} - 1) \mid m$. Let $m = d(2^{k+1} - 1)$, so d is divisor of m.

Then, $\sigma(m)=\dfrac{2^{k+1}m}{2^{k+1}-1}=m+\dfrac{m}{2^{k+1}-1}=m+d.$

But $\sigma(m)$ is defined as the sum of all of the divisors of m. Therefore, m only has the two divisors m and d, so m is prime, d is 1, and so $2^{k+1} - 1$ is prime. Hence, we have shown that all even perfect numbers are of the form $2^{n-1}(2^n - 1)$, where $2^n - 1$ is prime.

3. Here, there is no need that ΔABC be contained in one of the half planes determined by r.



If the coordinates of A are (x_1, y_1), the coordinates of B are (x_2, y_2) and those of C are (x_3, y_3) in a cartesian coordinate system with r as x-axis then A' has coordinates $(x_1, -y_1)$, B' has coordinates $(x_2, -y_2)$ and C' has coordinates $(x_3, -y_3)$.

The equation of the line through A' parallel to BC is $(y_2 - y_3)x - (x_2 - x_3)y - x_1(y_2 - y_3) - y_1(x_2 - x_3) = 0$

The equation of the line through B' parallel to CA is $(y_3 - y_1)x - (x_3 - x_1)y - x_2(y_3 - y_1) - y_2(x_2 - x_1) = 0$

The equation of the line through C' parallel to AB is $(y_1 - y_2)x - (x_1 - x_2)y - x_3(y_1 - y_2) - y_3(x_1 - x_2) = 0$

Since the three equations when added together vanish identically, the lines represented by them meet in a point. Its coordinates are found, by solving between any two, to be

$$\left(\frac{(x_1^2+x_2x_3)(y_2-y_3)+(x_2^2+x_3x_1)(y_3-y_1)+(x_3^2+x_1x_2)(y_1-y_2)}{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)},\right.$$

$$\left.\frac{(y_1^2+y_2y_3)(x_2-x_3)+(y_2^2+y_3y_1)(x_3-x_1)+(y_3^2+y_1y_2)(x_1-x_2)}{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)}\right)$$

4. The squares can only be 0, 1, 4 or 7 mod 9. Thus the sum of the digits of a perfect square cannot be 2, 3, 5, 6 or 8 mod 9, since the number itself would then be 2, 3, 5, 6 or 8 mod 9.

We shall show that the sum of the digits of a perfect square can take every value of the form 0, 1, 4 or 7 mod 9.

$$(10^m-1)^2=10^{2m}-2\cdot10^m+1=9\,\underbrace{9\ldots9}_{m-1}\,8\,\underbrace{0\ldots0}_{m-1}\,1,\; m\geq1.$$

The sum of the digits is $9m$, giving all the values greater than or equal to 9 congruent to 0 mod 9

$$(10^m-2)^2=10^{2m}-4\cdot10^m+4=9\,\underbrace{9\ldots9}_{m-1}\,6\,\underbrace{0\ldots0}_{m-1}\,4,\; m\geq1.$$

The sum of the digits is $9m + 1$, which gives all values greater than or equal to 10 congruent to 1 mod 9

$$(10^m-3)^2=10^{2m}-6\cdot10^m+9=9\,\underbrace{9\ldots9}_{m-1}\,4\,\underbrace{0\ldots0}_{m-1}\,9,\; m\geq1.$$

The sum of the digits is $9m + 4$, which takes every value greater than or equal to 13 which is congruent to 4 mod 9

$$(10^m-5)^2=10^{2m}-10^{m+1}+25=\underbrace{9\ldots9}_{m-1}\,0\,\underbrace{0\ldots0}_{m-1}\,2\,5.$$

SAMURAI SUDOKU

ANSWER - OCTOBER 2019

4	3	1	7	5	6	9	8	2				4	2	8	1	6	9	7	5	3
6	5	7	9	2	8	1	4	3				7	3	9	4	5	8	2	6	1
2	8	9	3	1	4	7	5	6				1	6	5	2	7	3	8	4	9
8	6	3	2	7	5	4	9	1				6	7	2	8	9	4	3	1	5
5	9	4	8	3	1	6	2	7				8	5	3	6	1	7	9	2	4
7	1	2	4	6	9	5	3	8				9	4	1	3	2	5	6	8	7
3	4	5	1	8	7	2	6	9	4	5	1	3	8	7	5	4	6	1	9	2
1	2	6	5	9	3	8	7	4	9	3	2	5	1	6	9	3	2	4	7	8
9	7	8	6	4	2	3	1	5	6	8	7	2	9	4	7	8	1	5	3	6
						1	9	3	7	6	5	4	2	8						
						6	2	7	3	4	8	9	5	1						
						5	4	8	1	2	9	6	7	3						
6	1	9	3	7	8	4	5	2	8	7	3	1	6	9	3	8	2	5	7	4
8	4	5	6	9	2	7	3	1	2	9	6	8	4	5	7	6	9	3	2	1
2	7	3	1	4	5	9	8	6	5	1	4	7	3	2	5	4	1	8	9	6
5	2	4	9	1	7	8	6	3				5	7	1	8	3	6	2	4	9
7	8	1	2	3	6	5	9	4				6	9	3	4	2	7	1	5	8
9	3	6	5	8	4	1	2	7				4	2	8	9	1	5	7	6	3
4	9	2	7	5	3	6	1	8				2	5	4	1	9	8	6	3	7
3	5	8	4	6	1	2	7	9				9	8	6	2	7	3	4	1	5
1	6	7	8	2	9	3	4	5				3	1	7	6	5	4	9	8	2

The sum of the digits is $9(m-1)+7=9m-2$, from which we get every value greater than or equal to 7 congruent to 7 mod 9.

Note : We have taken care of all the integers apart from 0, 1, 4, which are the sums of the digits of 0^2, 1^2 and 2^2 respectively.

5. Let the four marks be $10a+b$, $10c+d$, $10e+f$ and $10g+h$.

Then, $10a+b+10c+d+10e+f+10g+h$

$= 10b+a+10d+c+10f+e+10h+g$,

So, $9(a+c+e+g)=9(b+d+f+h)$

and $a+c+e+g=b+d+f+h$. ...(i)

Therefore, the average of the four marks is

$$\frac{10a+b+10b+a+10c+d+10d+c+10e+f+10f+e+10g+h+10h+g}{4\cdot 2}$$

$$=\frac{11(a+b+c+d+e+f+g+h)}{8} \quad \text{...(ii)}$$

The average must consist only of k's and 0's, the digits not equal to a, b, c, d, e, f, g or h.

Thus, from (ii) $\frac{11(1+2+...+9-k)}{8}=\frac{11(45-k)}{8}$,

and so $8 \mid (45-k)$. Since $k \le 9$, the only solution for k is $k=5$. Therefore, Sam's average is $11(40)/8=55$.

6. We know from Cauchy's inequality that

$$\left(\sum_{i=1}^{n}(a_i-b_i)\right)^2 \le \left(\sum_{i=1}^{n}(a_i+b_i)\right)\left(\sum_{i=1}^{n}\frac{(a_i-b_i)^2}{a_i+b_i}\right)$$

Therefore

$$\left(\sum_{i=1}^{n}a_i\right)\left(\sum_{i=1}^{n}b_i\right)=\frac{1}{4}\left(\left(\sum_{i=1}^{n}a_i+\sum_{i=1}^{n}b_i\right)^2-\left(\sum_{i=1}^{n}a_i-\sum_{i=1}^{n}b_i\right)^2\right)$$

$$=\frac{1}{4}\left(\left(\sum_{i=1}^{n}(a_i+b_i)\right)^2-\left(\sum_{i=1}^{n}(a_i-b_i)\right)^2\right)$$

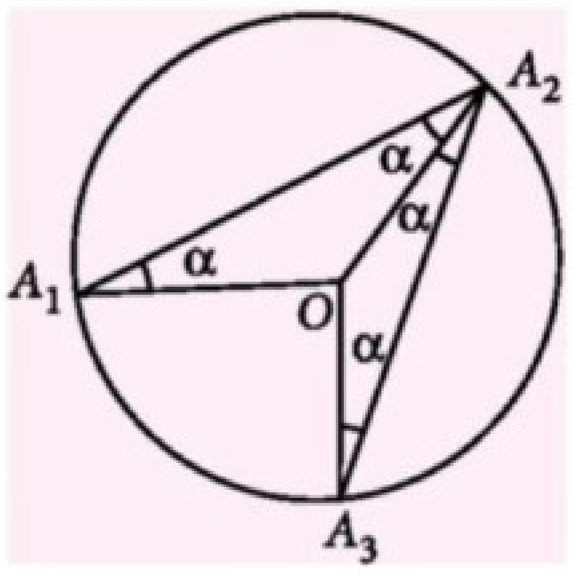
$$\ge\frac{1}{4}\left(\left(\sum_{i=1}^{n}(a_i+b_i)\right)^2-\left(\sum_{i=1}^{n}(a_i+b_i)\right)\left(\sum_{i=1}^{n}\frac{(a_i-b_i)^2}{a_i+b_i}\right)\right)$$

$$=\left(\sum_{i=1}^{n}(a_i+b_i)\right)\left(\sum_{i=1}^{n}\frac{(a_i+b_i)^2-(a_i-b_i)^2}{4(a_i+b_i)}\right)$$

$$=\left(\sum_{i=1}^{n}(a_i+b_i)\right)\left(\sum_{i=1}^{n}\frac{a_ib_i}{a_i+b_i}\right)$$

which completes the proof.

7. We let the ball start at A_1 as shown. Then $\angle OA_1A_2 = \angle A_1A_2O = \alpha$ (equal radii) and $\angle A_1OA_2 = 180° - 2\alpha$ (angles sum to 180°).



Since the angle of incidence equals the angle of reflection, we have $\angle A_3A_2O = \angle A_1A_2O = \alpha = \angle A_2A_3O$ by equal radii. So, $\angle A_3OA_2 = \angle A_2OA_1 = 180° - 2\alpha$. Thus, the angles at the centre are always equal. The cue ball returns to A_1 iff

$k(180° - 2\alpha) = 360°c \Rightarrow k(90° - \alpha) = 180°c$,

for some positive integers c and k. Since α must be real, $90° - \alpha$ must be either a rational or irrational number.

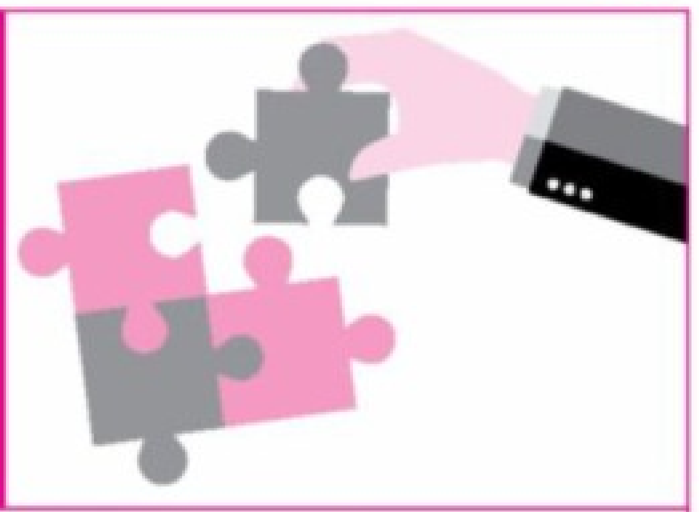
Case I: If $90° - \alpha$ is rational, it can be expressed in the from $\frac{m}{n}$, where m and n are positive, relatively prime integers. Thus $k\cdot\frac{m}{n}=180°c$. We may take $k = 180°n$ and then $m = c$, so for all α rational, the cue ball will return to A_1.

Case II: If $90° - \alpha$ is irrational, let $x = 90° - \alpha$. Then $kx = 180° \Rightarrow x=\frac{180°c}{k}$.

But x is irrational, so we have a contradiction.

❖❖

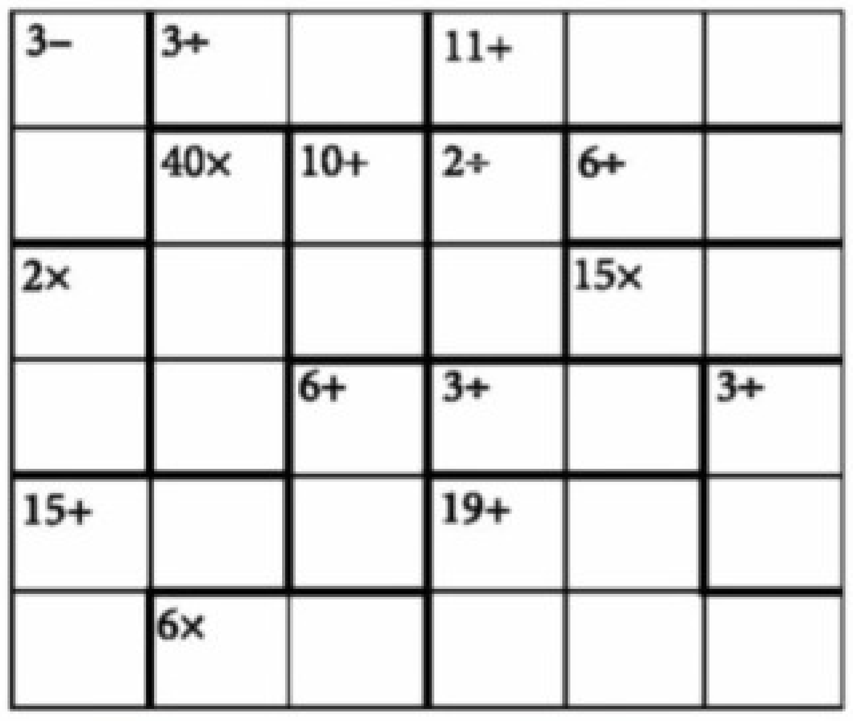
PUZZLE CORNER



MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics. In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is combined using a specified operation on its numbers. For example, if that value is 3 for a two-box cluster and operation is multiply, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.



3–	3+		11+		
	40×	10+	2÷	6+	
2×				15×	
		6+	3+		3+
15+			19+		
	6×				

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

FOCUS

Between 6th to 11th January

JEE MAIN 2020

PRACTICE PAPER

with Numerical Value Type Questions

1. Domain of definition of the function $f(x)=\sqrt{\sin^{-1}(2x)+\frac{\pi}{6}}$ for real valued x, is

(a) $\left[-\frac{1}{4},\frac{1}{2}\right]$ (b) $\left[-\frac{1}{2},\frac{1}{2}\right]$

(c) $\left(-\frac{1}{2},\frac{1}{9}\right)$ (d) $\left[-\frac{1}{4},\frac{1}{4}\right]$

2. The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is

(a) 2 (b) 1 (c) 4 (d) 3

3. Let $f(a) = g(a) = k$ and their n^{th} derivatives $f^n(a), g^n(a)$ exist and are not equal for some n. Further if $\lim\limits_{x\to a}\frac{f(a)g(x)-f(a)-g(a)f(x)+g(a)}{g(x)-f(x)} = 4$, then the value of k is

(a) 2 (b) 1 (c) 0 (d) 4

4. If $\alpha, \beta \neq 0$ and $f(n) = \alpha^n + \beta^n$ and

$$\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix}$$

$= K(1 - \alpha)^2 (1 - \beta)^2(\alpha - \beta)^2$, then K is equal to

(a) $\frac{1}{\alpha\beta}$ (b) 1 (c) −1 (d) $\alpha\beta$

5. Let 10 vertical poles standing at equal distances on a straight line, subtend the same angle of elevation α at a point O on this line and all the poles are on the same side of O. If the height of the longest pole is 'h' and the distance of the foot of the smallest pole from O is 'a'; then the distance between two consecutive poles is

(a) $\frac{h\sin\alpha+a\cos\alpha}{9\sin\alpha}$ (b) $\frac{h\cos\alpha-a\sin\alpha}{9\cos\alpha}$

(c) $\frac{h\cos\alpha-a\sin\alpha}{9\sin\alpha}$ (d) $\frac{h\sin\alpha+a\cos\alpha}{9\cos\alpha}$

6. The area of the region bounded by the parabola $(y - 2)^2 = x - 1$, the tangent to the parabola at the point (2, 3) and the x-axis is

(a) 6 (b) 9 (c) 12 (d) 3

7. Let I be the purchase value of an equipment and $V(t)$ be the value after it has been used for t years. The value $V(t)$ depreciates at a rate given by differential equation $\frac{dV(t)}{dt}=-k(T-t)$; where $k > 0$ is a constant and T is the total life in years of the equipment. Then the scrap value $V(T)$ of the equipment is

(a) $I-\frac{k(T-t)^2}{2}$ (b) e^{-kT}

(c) $T^2-\frac{I}{k}$ (d) $I-\frac{kT^2}{2}$

8. If m is A. M. of two distinct real numbers l and n $(l, n > 1)$ and G_1, G_2 and G_3 are three geometric means between l and n, then $G_1^4+2G_2^4+G_3^4$ equals

(a) $4\,lmn^2$ (b) $4l^2m^2n^2$ (c) $4\,l^2mn$ (d) $4\,lm^2n$

9. If z_1 and z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then $\arg z_1 - \arg z_2$ is equal to

(a) $-\pi$ (b) $\pi/2$ (c) $-\pi/2$ (d) 0

10. If the coefficient of x^7 in $\left[ax^2 + \left(\frac{1}{bx}\right)\right]^{11}$ equals the coefficient of x^{-7} in $\left[ax - \left(\frac{1}{bx^2}\right)\right]^{11}$, then a and b satisfy the relation

(a) $a + b = 1$ (b) $a - b = 1$

(c) $ab = 1$ (d) $\frac{a}{b} = 1$

11. Let k be a non-zero real number. If

$$f(x) = \begin{cases} \dfrac{(e^x - 1)^2}{\sin\left(\dfrac{x}{k}\right)\log\left(1 + \dfrac{x}{4}\right)}, & x \neq 0 \\ 12, & x = 0 \end{cases}$$

is a continuous function, then the value of k is

(a) 1 (b) 2 (c) 3 (d) 4

12. Three distinct points A, B and C are given in the 2-dimensional coordinate plane such that the ratio of the distance of any one of them from the point (1, 0) to the distance from the point (–1, 0) is equal to 1/3. Then the circumcentre of the triangle ABC is at the point

(a) $\left(\frac{5}{4}, 0\right)$ (b) $\left(\frac{5}{2}, 0\right)$ (c) $\left(\frac{5}{3}, 0\right)$ (d) (0, 0)

13. If the shortest distance between the lines $\frac{x-1}{\alpha} = \frac{y+1}{-1} = \frac{z}{1}, (\alpha \neq -1)$ and $x + y + z + 1 = 0 = 2x - y + z + 3$ is $\frac{1}{\sqrt{3}}$, then a value of α is

(a) $-\frac{16}{19}$ (b) $-\frac{19}{16}$ (c) $\frac{32}{19}$ (d) $\frac{19}{32}$

14. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house is

(a) 1/9 (b) 2/9 (c) 7/9 (d) 8/9

15. The distance, from the origin, of the normal to the curve, $x = 2\cos t + 2t \sin t$, $y = 2\sin t - 2t \cos t$ at $t = \pi/4$ is

(a) 4 (b) $2\sqrt{2}$ (c) 2 (d) $\sqrt{2}$

16. Let $f(x)$ be a polynomial function of second degree. If $f(1) = f(-1)$ and a, b, c are in A.P., then $f'(a)$, $f'(b)$ and $f'(c)$ are in

(a) G.P.

(b) H.P.

(c) A.P.

(d) None of these

17. For the hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$, which of the following remains constant when α varies ?

(a) abscissa of vertices (b) abscissa of foci

(c) eccentricity (d) directrix

18. If the plane $2ax - 3ay + 4az + 6 = 0$ passes through the midpoint of the line joining the centres of the spheres $x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$ and $x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$ then a equals

(a) 1 (b) –1 (c) 2 (d) –2

19. Consider :

Statement-1 : $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is a fallacy.

Statement-2 : $(p \to q) \leftrightarrow (\sim q \to \sim p)$ is a tautology.

(a) Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is false.

(c) Statement-1 is false, Statement-2 is true.

(d) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1.

20. Let $ABCD$ be a parallelogram such that $\overrightarrow{AB} = \vec{q}, \overrightarrow{AD} = \vec{p}$ and $\angle BAD$ be an acute angle. If $\vec{r}$ is the vector that coincides with the altitude directed from the vertex B to the side AD, then $\vec{r}$ is given by

(a) $\vec{r} = \vec{q} - \left(\frac{\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}\right)\vec{p}$ (b) $\vec{r} = -3\vec{q} + \frac{3(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$

(c) $\vec{r} = 3\vec{q} - \frac{3(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$ (d) $\vec{r} = -\vec{q} + \left(\frac{\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}\right)\vec{p}$

Numerical Value Type

21. If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3×3 matrix A and $|A| = 4$, then α is equal to

22. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is

23. Let $\cos(\alpha + \beta) = \frac{4}{5}$ and $\sin(\alpha - \beta) = \frac{5}{13}$, where $0 \leq \alpha, \beta \leq \frac{\pi}{4}$. Then tan2α is equal to

24. Let $f : R \to R$ be a continuous odd function, which vanishes exactly at one point and $f(1) = \frac{1}{2}$. Suppose that $F(x) = \int_{-1}^{x} f(t)dt$ for all $x \in [-1, 2]$

and $G(x)=\int_{-1}^{x} t\,|f(f(t))|\,dt$ for all $x \in [-1, 2]$. If $\lim_{x\to 1}\frac{F(x)}{G(x)}=\frac{1}{14}$, then the value of $f\left(\frac{1}{2}\right)$ is

25. If the mean deviation of numbers 1, $1 + d$, $1 + 2d$, ..., $1 + 100d$ from their mean is 255, then d is equal to

SOLUTIONS

1. (a) : For $f(x)=\sqrt{\sin^{-1}(2x)+\frac{\pi}{6}}$

to be defined and real, $\sin^{-1} 2x + \pi/6 \geq 0$

$\Rightarrow \sin^{-1} 2x \geq -\frac{\pi}{6}$ ...(i)

But we know that $-\pi/2 \leq \sin^{-1} 2x \leq \pi/2$ ...(ii)

Combining (i) and (ii), $-\pi/6 \leq \sin^{-1} 2x \leq \pi/2$

$\Rightarrow \sin(-\pi/6) \leq 2x \leq \sin(\pi/2)$

$\Rightarrow -1/2 \leq 2x \leq 1 \Rightarrow -1/4 \leq x \leq 1/2$

$\therefore D_f=\left[-\frac{1}{4}, \frac{1}{2}\right]$.

2. (a) : Let α and 4β be the roots of $x^2 - 6x + a = 0$ and α and 3β be the roots of the equation $x^2 - cx + 6 = 0$. From the relation between roots and coefficients

$\alpha + 4\beta = 6$ and $4\alpha\beta = a$

$\alpha + 3\beta = c$ and $3\alpha\beta = 6$,

we obtain $\alpha\beta = 2$ giving $a = 8$

The first equation is $x^2 - 6x + 8 = 0 \Rightarrow x = 2, 4$

For $\alpha = 2$, $4\beta = 4 \Rightarrow 3\beta = 3$

For $\alpha = 4$, $4\beta = 2 \Rightarrow 3\beta = 3/2$ (not an integer)

So the common root is $\alpha = 2$.

3. (d) : $\displaystyle\lim_{x\to a}\frac{f(a)\,g(x)-f(a)-g(a)f(x)+g(a)}{g(x)-f(x)}=4$

$\Rightarrow \displaystyle\lim_{x\to a}\frac{f(a)[g(x)-f(x)]}{g(x)-f(x)}=4 \Rightarrow \lim_{x\to a} f(a)=4$

$\Rightarrow k = 4$

4. (b) : $\begin{vmatrix} 3 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$

$$=\begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}\begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \end{vmatrix}$$

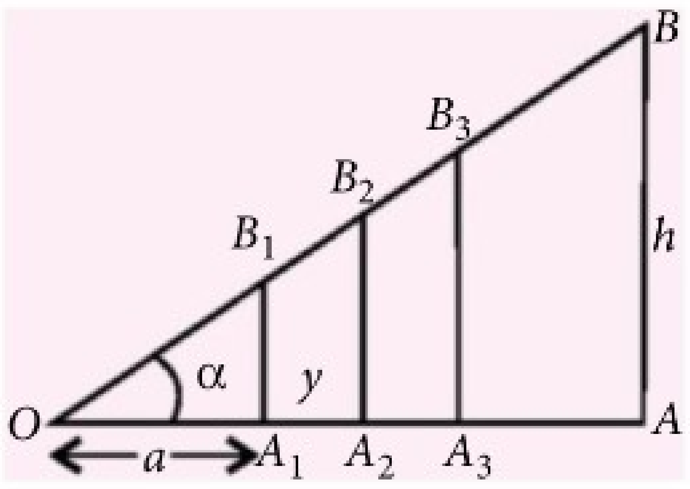
(By multiplication of determinants)

$= [(1 - \alpha)(1 - \beta)(\beta - \alpha)]^2$

On comparison, $K = 1$

5. (c) : Let AB be the longest pole of height h and x be the distance between consecutive poles.

$\therefore \Delta ABO \sim \Delta A_1B_1O$



$\therefore \frac{h}{a+9x}=\frac{y}{a}$

$\Rightarrow \frac{h}{a+9x}=\frac{a\tan\alpha}{a}$

$\Rightarrow a+9x=\frac{h}{\tan\alpha}$

$\Rightarrow x=\frac{h-a\tan\alpha}{9\tan\alpha}=\frac{(h\cos\alpha-a\sin\alpha)}{9\sin\alpha}$

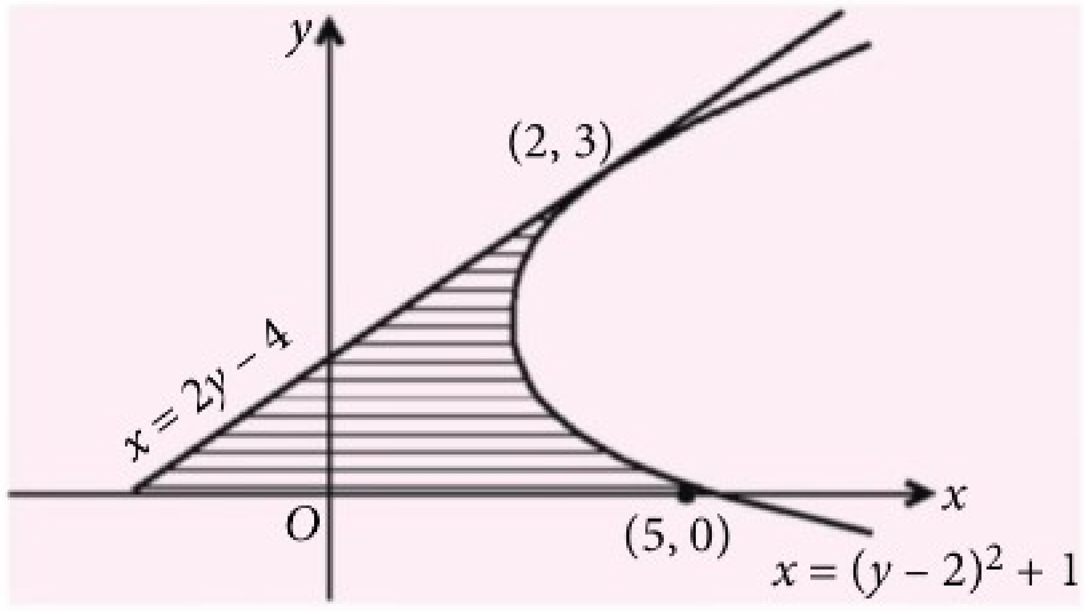
6. (b) : $(y - 2)^2 = x - 1$

Differentiating w.r.t. x, we have $2(y - 2)y' = 1$

$\Rightarrow y'=\frac{1}{2(y-2)}$. At $(2, 3)$, $y' = 1/2$

The equation of the tangent to the parabola at $(2, 3)$ is

$y - 3 = \frac{1}{2}(x - 2) \Rightarrow x - 2y + 4 = 0$



The area of the bounded region

$=\int_0^3[(y-2)^2+1-(2y-4)]\,dy$

$=\int_0^3(y^2-6y+9)\,dy=\int_0^3(y-3)^2\,dy$

Put $t = 3 - y \Rightarrow dt = -dy$

$\therefore$ Required area $=\int_0^3 t^2dt=\left[\frac{t^3}{3}\right]_0^3=\frac{3^3}{3}=9$

7. (d) : We have, $\frac{dV}{dt}=-k(T-t)$

On integration, $V=\frac{k(T-t)^2}{2}+\alpha$

At $t = 0$, $V(t) = I \Rightarrow I=\frac{kT^2}{2}+\alpha \quad \therefore \alpha=I-\frac{kT^2}{2}$

At $t = T$, we have $V(T) = \alpha = I-\frac{kT^2}{2}$

8. (d) : Given that l, G_1, G_2, G_3, n are in G.P.

$\therefore G_1 = lr$, $G_2 = lr^2$, $G_3 = lr^3$, $n = lr^4$

Then $G_1^4 + 2G_2^4 + G_3^4 = (lr)^4 + 2(lr^2)^4 + (lr^3)^4$

$= (l^3)(lr^4) + 2l^2(lr^4)^2 + l\,(lr^4)^3$
$= l^3 \times n + 2l^2 \times n^2 + ln^3 = ln(l^2 + 2nl + n^2)$
$= ln(n + l)^2 = 4m^2nl$

9. (d) : Let $z_1 = \cos\theta_1 + i\sin\theta_1$,
and $z_2 = \cos\theta_2 + i\sin\theta_2$
$\therefore\quad z_1 + z_2 = (\cos\theta_1 + \cos\theta_2) + i(\sin\theta_1 + \sin\theta_2)$
Now, $|z_1 + z_2| = |z_1| + |z_2|$
$\Rightarrow \sqrt{(\cos\theta_1+\cos\theta_2)^2+(\sin\theta_1+\sin\theta_2)^2} = 1+1$
$\Rightarrow\quad 2(1 + \cos(\theta_1 - \theta_2)) = 4$ (By squaring)
$\Rightarrow\quad \cos(\theta_1 - \theta_2) = 1 \Rightarrow \theta_1 - \theta_2 = 0$ ($\because \cos 0° = 1$)
$\Rightarrow\quad \arg z_1 - \arg z_2 = 0$

10. (c) : T_{r+1} of $\left(ax^2+\frac{1}{bx}\right)^{11} = {}^{11}C_r(ax^2)^r\left(\frac{1}{bx}\right)^{11-r}$

T_{r+1} of $\left(ax-\frac{1}{bx^2}\right)^{11} = {}^{11}C_r(ax)^r\left(-\frac{1}{bx^2}\right)^{11-r}$

$\therefore$ Coefficient of x^7 in $\left(ax^2+\frac{1}{bx}\right)^{11} = {}^{11}C_6\frac{a^6}{b^5}$

and coefficient of x^{-7} in $\left(ax-\frac{1}{bx^2}\right)^{11} = {}^{11}C_5\frac{a^5}{b^6}$

Now, ${}^{11}C_6\frac{a^6}{b^5} = {}^{11}C_5\frac{a^5}{b^6} \Rightarrow ab = 1$

11. (c) : Since $f(x)$ is a continuous function
$\therefore \lim\limits_{x\to 0} f(x) = f(0)$

$$\Rightarrow \lim_{x\to 0}\frac{\dfrac{(e^x-1)^2}{x^2}}{\dfrac{\sin(x/k)\cdot\log(1+x/4)}{k\cdot\dfrac{x}{k}\cdot 4\cdot\dfrac{x}{4}}} = 12$$

$\Rightarrow\quad 4k = 12 \quad\Rightarrow\quad k = 3$

12. (a) : Let P be a general point (x, y) such that
$\frac{PM}{PN} = \frac{1}{3}$ where $M \equiv (1, 0)$ and $N \equiv (-1, 0)$.

We have, $\frac{\sqrt{(x-1)^2+y^2}}{\sqrt{(x+1)^2+y^2}} = \frac{1}{3}$
$\Rightarrow\quad 9[(x - 1)^2 + y^2] = (x + 1)^2 + y^2$
$\Rightarrow\quad 8x^2 + 8y^2 - 20x + 8 = 0 \quad\Rightarrow\quad x^2+y^2-\frac{5}{2}x+1 = 0$

Thus, the locus is a circle with centre (5/4, 0).
As points A, B, C lie on this circle, the circumcentre of triangle ABC is (5/4, 0).

13. (c) : We have,
$x + y + z + 1 = 0,\ 2x - y + z + 3 = 0$ ... (i)
$\therefore$ Point of intersection of above lines is $P(0, 1, -2)$.

Given equation of line is $\frac{x-1}{\alpha} = \frac{y+1}{-1} = \frac{z}{1}$ ...(ii)
$\therefore$ Point Q (1, –1, 0) lies on above line.
$\therefore\ \overrightarrow{PQ} = \hat{i} - 2\hat{j} + 2\hat{k}$

Also, $\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 2 & -1 & 1\end{vmatrix} = 2\hat{i}+\hat{j}-3\hat{k}$ ...(iii) (From (i))

Now, $\vec{n} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \alpha & -1 & 1\\ 2 & 1 & -3\end{vmatrix}$ (From (ii) and (iii))

Shortest distance between lines = S. D. = $\overrightarrow{PQ}\cdot\hat{n}$
$= \frac{2-2(3\alpha+2)+2(\alpha+2)}{\sqrt{4+(3\alpha+2)^2+(\alpha+2)^2}} = \frac{1}{\sqrt{3}}$
$\Rightarrow\quad 3(2 - 4\alpha)^2 = 10\alpha^2 + (16\alpha + 12)$
$\Rightarrow\quad 19\alpha^2 - 32\alpha = 0 \Rightarrow \alpha = \frac{32}{19}$

14. (a) : No. of houses = 3 = No. of favourable cases
No. of applicants = 3
$\therefore$ Total number of events = 3^3
(because each candidate can apply in 3 ways)
$\therefore$ Required probability $= \frac{3}{3^3} = \frac{1}{9}$.

15. (c) : Given that $x = 2\cos t + 2t\sin t$
$\Rightarrow\quad \frac{dx}{dt} = -2\sin t + 2[t\cos t+\sin t] = 2t\cos t$ ...(i)
Also, $y = 2\sin t - 2t\cos t$
$\Rightarrow\quad \frac{dy}{dt} = 2\cos t - 2[-t\sin t+\cos t] = 2t\sin t$ ...(ii)
So, $\frac{dy}{dx} = \frac{2t\sin t}{2t\cos t} \Rightarrow \frac{dy}{dx} = \tan t$ (From (i) and (ii))
$\therefore\quad \left(\frac{dy}{dx}\right)_{t=\pi/4} = 1$
So, the slope of the normal is –1.
and at $t = \pi/4$, $x = \sqrt{2}+\frac{\pi}{2\sqrt{2}}$ and $y = \sqrt{2}-\frac{\pi}{2\sqrt{2}}$
$\therefore$ The equation of normal is
$$\left[y-\left(\sqrt{2}-\frac{\pi}{2\sqrt{2}}\right)\right] = -1\left[x-\left(\sqrt{2}+\frac{\pi}{2\sqrt{2}}\right)\right]$$
$\Rightarrow\quad y-\sqrt{2}+\frac{\pi}{2\sqrt{2}} = -x+\sqrt{2}+\frac{\pi}{2\sqrt{2}}$
$\Rightarrow\quad x + y = 2\sqrt{2}$
So, the distance from the origin is 2.

16. (c) : Let the polynomial be $f(x) = ax^2 + bx + c$

given $f(1) = f(-1) \Rightarrow b = 0$

$\therefore \quad f(x) = ax^2 + c$

Now $f'(x) = 2ax$

$\therefore \quad f'(a) = 2a^2, f'(b) = 2ab, f'(c) = 2ac$

As $a, b, c \in$ A.P.

$\Rightarrow \quad a^2, ab, ac \in$ A.P. $\Rightarrow 2a^2, 2ab, 2ac \in$ A.P.

$\Rightarrow \quad f'(a), f'(b), f'(c) \in$ A.P.

17. (b) : $b^2 = a^2(e^2 - 1) \Rightarrow \sin^2\alpha = \cos^2\alpha\,(e^2 - 1)$

$\Rightarrow \quad \tan^2\alpha + 1 = e^2 \Rightarrow e^2 = \sec^2\alpha$

Vertices $\equiv (\pm\cos\alpha, 0)$

Coordinates of foci are $(\pm ae, 0) \equiv (\pm 1, 0)$

$\Rightarrow$ If α varies then the abscissa of foci remains constant.

18. (d) : Centre of spheres are (–3, 4, 1) and (5, –2, 1). So, the coordinates of midpoint of line joining the centres are $\left(\frac{5-3}{2}, \frac{4-2}{2}, \frac{1+1}{2}\right)$ *i.e.*, (1, 1, 1)

Using mid point in the equation

$2ax - 3ay + 4az + 6 = 0$

$\Rightarrow \quad 2a - 3a + 4a + 6 = 0 \Rightarrow a = -2$

19. (a) : Let's prepare the truth table for the statements.

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$\sim p \wedge q$	$(p \wedge \sim q) \wedge (\sim p \wedge q)$
T	T	F	F	F	F	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	F	F

So, Statement-1 is fallacy.

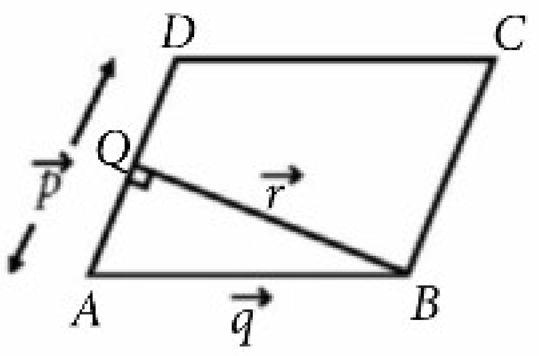
p	q	$\sim p$	$\sim q$	$p \to q$	$\sim q \to p$	$(p \to q) \to (\sim q \to p)$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

So, Statement-2 is tautology.

20. (d) : $\vec{r} = \overrightarrow{BA} + \overrightarrow{AQ}$

$= -\vec{q} +$ projection of $\overrightarrow{BA}$ across $\overrightarrow{AD}$

$$= -\vec{q} + \left(\frac{\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}\right)\vec{p}$$



21. (11) : $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$

det $P = 1(12 - 12) - \alpha(4 - 6) + 3(4 - 6) = 2\alpha - 6$

Also, det(adj P) = (det P)^2

$\Rightarrow \quad 2\alpha - 6 = 16 \Rightarrow \quad 2\alpha = 22 \quad \therefore \quad \alpha = 11$

Remark : det(adj A) = (det A)$^{n-1}$, where A is a matrix of order n.

22. (879) : Number of ways in which one or more balls can be selected from 10 white, 9 green, 7 black balls is

= (10 + 1) (9 + 1) (7 + 1) – 1 = 880 – 1 = 879 ways

23. (1.69) : $\cos(\alpha + \beta) = 4/5$ gives $\tan(\alpha + \beta) = 3/4$

Also $\sin(\alpha - \beta) = 5/13$ gives $\tan(\alpha - \beta) = 5/12$

$\tan(2\alpha) = \tan(\alpha + \beta + \alpha - \beta)$

$$= \frac{\tan(\alpha+\beta) + \tan(\alpha-\beta)}{1 - \tan(\alpha+\beta)\tan(\alpha-\beta)} = \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4}\times\frac{5}{12}} = \frac{56}{33} = 1.69$$

24. (7) : $\lim\limits_{x\to 1}\frac{F(x)}{G(x)} = \lim\limits_{x\to 1}\frac{F'(x)}{G'(x)} = \lim\limits_{x\to 1}\frac{f(x)}{x\,|f(f(x))|} = \frac{1}{14}$

$$= \frac{f(1)}{\left|f\left(\frac{1}{2}\right)\right|} = \frac{1}{14}$$

Thus $\left|f\left(\frac{1}{2}\right)\right| = 7$. But $f\left(\frac{1}{2}\right) \neq -7$ as $f(x)$ vanishes exactly at one point.

25. (10.1) : The numbers are 1, 1 + d, 1 + 2d, ..., 1 + 100d.

The numbers are in A.P.

Then mean = 51st term = 1 + 50d = (say)

Mean deviation (M.D.) $= \frac{1}{n}\sum\limits_{i=1}^{101}|x_i - \bar{x}|$

$$= \frac{1}{101}[50d + 49d + 48d + \ldots. + d + 0 + d + 2d + \ldots + 50d]$$

$$= \frac{1}{101}\cdot 2d(1 + 2 + \ldots. + 50) = \frac{1}{101}\cdot 2d\cdot\frac{50\cdot 51}{2} = \frac{50\cdot 51}{101}d$$

But M.D. = 255 (given)

$$\Rightarrow \quad \frac{50\cdot 51}{101}d = 255 \;\Rightarrow\; d = \frac{101\times 255}{50\times 51} = \frac{101\times 255}{2550} = 10.1$$

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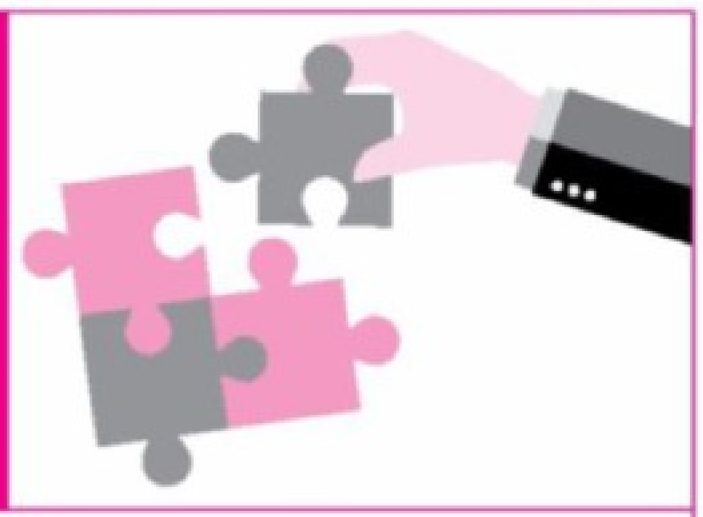
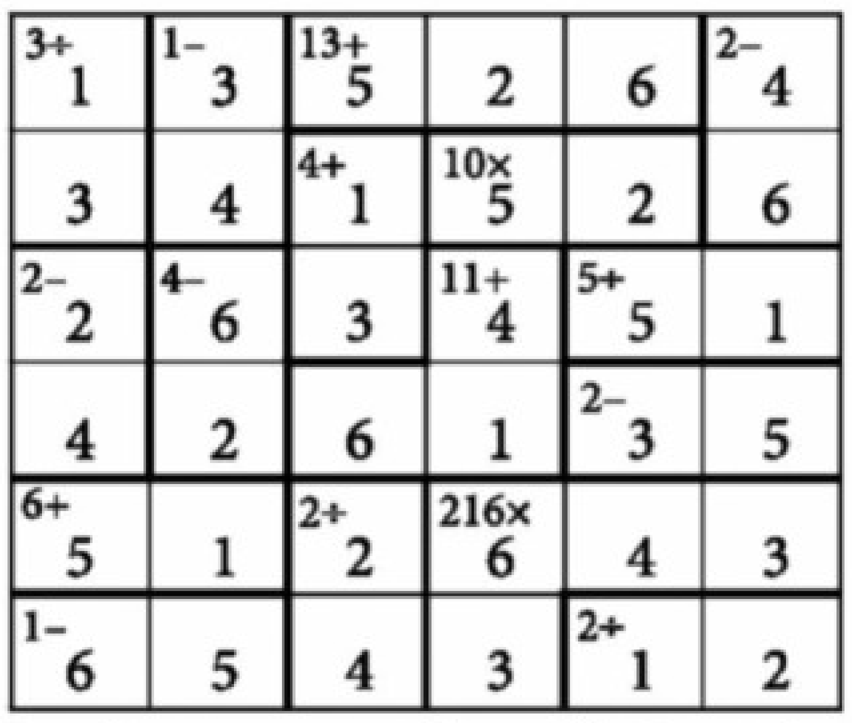
PUZZLE CORNER

ANSWER - OCTOBER 2019





3+ 1	1– 3	13+ 5	2	6	2– 4
3	4	4+ 1	10× 5	2	6
2– 2	4– 6	3	11+ 4	5+ 5	1
4	2	6	1	2– 3	5
6+ 5	1	2+ 2	216× 6	4	3
1– 6	5	4	3	2+ 1	2

Consider 4– in place of 14+ occurring in 3rd row 2nd column in the October puzzle.

MATHS MUSING

SOLUTION SET-202

1. (c) : $x^2 - (y+42)^2 = 2012 - 42^2 = 248 = 124 \times 2$

$\therefore\ x + y + 42 = 124,\ x - y - 42 = 2$

$\Rightarrow\ x = 63,\ y = 19\ \therefore\ x - 3y = 6$

2. (b) : $M = \sum_{s=1}^{10} \sum_{r=0}^{s-1} \left(2^s - 2^r\right)$

$= \sum_{s=1}^{10} [s \cdot 2^s - (1 + 2 + 2^2 + \ldots + 2^{s-1})] = \sum_{s=1}^{10} \left((s-1)2^s + 1\right)$

$= 4\,(1 + 2{\cdot}2 + 3{\cdot}2^2 + \ldots + 9{\cdot}2^8) + 10 = 14 + 8{\cdot}2^{11}$, summing A.G.P. $= 14 + 2^{14} = 16398$

3. (a) : $S \equiv x^2 + y^2 = 81,\ S_1 \equiv (x+3)^2 + y^2 = 36$ and $S_2 \equiv (x-6)^2 + y^2 = 9$

If $ax + by = 1$ is a common tangent to S_1 and S_2.

Then, $|3a+1| = 6\sqrt{a^2+b^2},\ |6a-1| = 3\sqrt{a^2+b^2}$

After solving, $a = \frac{1}{15},\ b = \frac{2\sqrt{2}}{15}$.

So, the chord is $x + 2\sqrt{2}y = 15$. Its distance from the centre of S is 5.

$\therefore$ The length of the chord $= 2\sqrt{9^2 - 5^2} = 4\sqrt{14}$

4. (d) : H, T, X stand for head, tail, head or tail. The required sequence of atleast 8 consecutive heads is 8HXXXX, T8HXXX, XT8HXX, XXT8HX, XXXT8H.

Probability $= \frac{1}{2^8} + \frac{4}{2^9} = \frac{3}{2^8}$

5. (d) : $a = \frac{x^2+4}{|x|} - 3 = |x| + \frac{4}{|x|} - 3 \ge 2\sqrt{|x| \cdot \frac{4}{|x|}} - 3$

[$\because$ A.M. $\ge$ G.M.]

$\Rightarrow\ a \ge 4 - 3 = 1\ \Rightarrow\ a \ge 1$

6. (a,c,d) : Let $f(x) = 1 - x^2 + 2^x,\ f(-2) = -\frac{11}{4},\ f(-1) = \frac{1}{2}$

$\therefore\ f(x) = 0$ for $x \in (-2, -1), f(3) = 0, x \in (2, 3]$

$f'(x) = -2x + 2^x \ln 2, f'(3) = -6 + 8 \ln 2 < 0$

$\Rightarrow\ f(x) < 0$ for some $x \in (3, 4)$. But $f(4) = 1$

$\therefore\ f(x) = 0$ for $x \in (3, 4)$

7. (c) : The tangent to $y = f(x)$ at (x, f) meets the x-axis at $\left(\frac{f}{f'}, 0\right)$ ...(i)

The tangent to $y = g(x)$ at (x, g) meets the x-axis at $\left(\frac{g}{g'}, 0\right)$ ...(ii)

(i) and (ii) are same points $\therefore\ g = \frac{f^2}{f'}$ (since $g' = f$)

Differentiating, $ff'' = (f')^2 \Rightarrow \frac{f''}{f'} = \frac{f'}{f} \Rightarrow f' = cf, f = A\,e^{cx}$

$f(0) = 1 \Rightarrow f = e^{cx},\ g(x) = \int_{-\infty}^{x} e^{ct}\,dt = \frac{e^{cx}}{c}$,

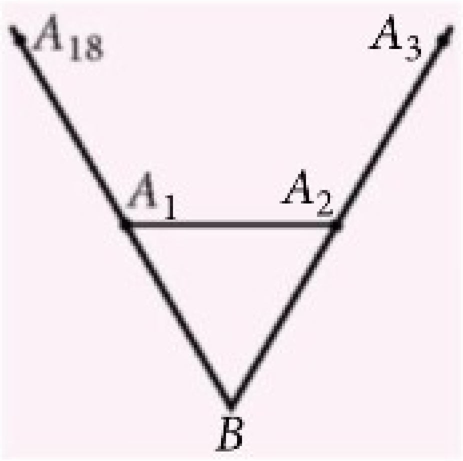
$g(0) = \frac{1}{2} \Rightarrow g(x) = \frac{e^{2x}}{2},\ f(x) = e^{2x} \quad \Rightarrow \quad f'(x) = 2e^{2x}$

$f'(0) = 2$, tangent at $(0, 1)$ is $2x - y + 1 = 0$, and normal is $x + 2y - 2 = 0$

The area bounded by these lines and x-axis is $\frac{5}{4}$.

8. (d) : Area $= \int_0^1 \left(g(x) - x\right) dx = \int_0^1 \left(\frac{e^{2x}}{2} - x\right) dx = \frac{e^2 - 3}{4}$

9. (9) : $2\pi = \angle A_{18}A_1B + \angle BA_1A_2 + \angle A_{18}A_1A_2$



$= \left(1 - \frac{2}{n}\right)\pi + \frac{\pi}{3} + \frac{8\pi}{9}$

$= \frac{20\pi}{9} - \frac{2\pi}{n}$

$\therefore\ n = 9$

10. (d) : P $\rightarrow$ 2 ; Q $\rightarrow$ 4 ; R $\rightarrow$ 1 ; S $\rightarrow$ 3

(P) x, y, z are in A.P. $\Rightarrow x + z = 2y$, even

$\therefore$ Both x and z are even or both are odd.

$\therefore$ The number of subsets is $\binom{5}{2} + \binom{5}{2} = 20$

(Q) The number of triples is $\binom{10-3+1}{3} = \binom{8}{3} = 56$

(R) 1 4 7 10
2 5 8
3 6 9

$x^3 + y^3$ is divisible by 3 if $x + y$ is divisible by 3.

$\therefore$ x and y belong to 3rd row or one from 1st row and the second from 2nd row.

$\therefore$ The number of subsets is $\binom{3}{2} + \binom{4}{1}\binom{3}{1} = 15$

(S) $x^2 - y^2 = (x+y)(x-y)$

$\therefore$ The number of subsets is $\binom{4}{2} + 2\binom{3}{2} + \binom{4}{1}\binom{3}{1} = 24$

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YOU ASK
WE ANSWER

Do you have a question that you just can't get answered?
Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. Solve the differential equation :

$$\frac{dy}{dx}+\frac{(ax+hy+g)}{(hx+by+f)}=0$$

(Soham Bose, West Bengal)

Ans. Given differential equation is

$$\frac{dy}{dx}+\frac{ax+hy+g}{hx+by+f}=0$$

$\Rightarrow$ $(hx + by + f)dy + (ax + hy + g)dx = 0$

$\Rightarrow$ $hx\,dy + by\,dy + f\,dy + ax\,dx + hy\,dx + gdx = 0$

$\Rightarrow$ $h(x\,dy + y\,dx) + by\,dy + f\,dy + ax\,dx + gdx = 0$

$\Rightarrow$ $h\,d(xy) + by\,dy + f\,dy + ax\,dx + gdx = 0$

On integrating both sides, we get

$hxy+b\frac{y^2}{2}+fy+a\frac{x^2}{2}+gx=c$, which is the required solution.

2. For $r = 0, 1, \ldots\ldots\ldots, 10$, let A_r, B_r, C_r denote respectively, the coefficients of x^r in the expansion of $(1 + x)^{10}$, $(1 + x)^{20}$, $(1 + x)^{30}$.

Then find the value of $\sum\limits_{r=1}^{10} A_r\,(B_{10}B_r - C_{10}\,A_r)$.

(Satyadev Kumar, Bihar)

Ans. $(1 + x)^{10} = A_0 + A_1x + A_2x^2 + \ldots + A_{10}x^{10}$

$(x + 1)^{20} = B_0x^{20} + B_1x^{19} + B_2x^{18} + \ldots + B_{20}$. Considering the coefficient of x^{20} in the product, we get

$A_0B_0 + A_1B_1 + A_2B_2 + \ldots + A_{10}\,B_{10}$ = coefficient of x^{20} in the expansion of $(1 + x)^{10}(x + 1)^{20} = (1 + x)^{30}$ which is C_{20}

$$\therefore \sum_{r=0}^{10} A_rB_r = C_{20} = C_{10}$$

$$\text{But } \binom{n}{0}^2+\binom{n}{1}^2+\binom{n}{2}^2+\ldots+\binom{n}{n}^2=\binom{2n}{n}$$

$$\therefore\ A_0^{\,2}+A_1^{\,2}+A_2^{\,2}+\ldots+A_{10}^{\,2}=\binom{20}{10}=B_{10}$$

$$\Rightarrow \sum_{r=0}^{10} A_r^{\,2} = B_{10}.$$

$$\text{Now, } \sum_{r=1}^{10} A_r\,(B_{10}\,B_r - C_{10}\,A_r)$$

$$= B_{10}\sum_{r=1}^{10} A_r\,B_r - C_{10}\sum_{r=1}^{10} A_r^{\,2}$$

$= B_{10}\,(C_{10} - 1) - C_{10}\,(B_{10} - 1) = C_{10} - B_{10}$.

3. Let ABC be an equilateral triangle inscribed in the circle $x^2 + y^2 = a^2$. Suppose perpendiculars from A, B, C to the major axis meet the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(a > b)$ respectively at P, Q, R so that P, Q, R lie on the same side of the major axis. Prove that the normals to the ellipse at P, Q, R are concurrent.

(Priyanshu Sharma, M.P.)

Ans. A, B, C are the vertices of an equilateral triangle.

$A\,(a\cos\theta,\,a\sin\theta)$, $B\left(a\cos\left(\theta+\frac{2\pi}{3}\right), a\sin\left(\theta+\frac{2\pi}{3}\right)\right)$ and

$C\left(a\cos\left(\theta+\frac{4\pi}{3}\right), a\sin\left(\theta+\frac{4\pi}{3}\right)\right)$

Hence, $P\,(a\cos\theta,\,b\sin\theta)$,

$Q\left(a\cos\left(\theta+\frac{2\pi}{3}\right), b\sin\left(\theta+\frac{2\pi}{3}\right)\right)$,

$R\left(a\cos\left(\theta+\frac{4\pi}{3}\right), b\sin\left(\theta+\frac{4\pi}{3}\right)\right)$

The normals to the ellipse at P, Q, R are respectively,

$$L_1 \equiv ax\sin\theta - by\cos\theta - \frac{a^2-b^2}{2}\sin 2\theta = 0;$$

$$L_2 \equiv ax\sin\left(\theta+\frac{2\pi}{3}\right) - by\cos\left(\theta+\frac{2\pi}{3}\right) - \frac{a^2-b^2}{2}\sin 2\left(\theta+\frac{2\pi}{3}\right) = 0;$$

$$L_3 \equiv ax\sin\left(\theta+\frac{4\pi}{3}\right) - by\cos\left(\theta+\frac{4\pi}{3}\right) - \frac{a^2-b^2}{2}\sin 2\left(\theta+\frac{4\pi}{3}\right) = 0$$

Since, $\sin\theta + \sin\left(\theta+\frac{2\pi}{3}\right) + \sin\left(\theta+\frac{4\pi}{3}\right) = 0$

$\cos\theta + \cos\left(\theta+\frac{2\pi}{3}\right) + \cos\left(\theta+\frac{4\pi}{3}\right) = 0$

$\sin 2\theta + \sin 2\left(\theta+\frac{2\pi}{3}\right) + \sin 2\left(\theta+\frac{4\pi}{3}\right) = 0$

Hence, L_1, L_2, L_3 are concurrent.

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CONCEPT MAP

STATISTICS

Class XI

STATISTICS

Measures of Dispersion

Mean Deviation

It is the arithmetic mean of the absolute values of deviations about some point (mean or median or mode).

$$\text{Mean Deviation} = \frac{\text{Sum of Deviations}}{\text{Number of Observations}}$$

For Ungrouped Data

Let $x_1, x_2, \ldots\ldots, x_n$ be n observations, then mean deviation about mean is given by M.D.$(\bar{x}) = \frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|$;

Mean deviation about median is given by,

M.D. $(M) = \frac{1}{n}\sum_{i=1}^{n}|x_i - M|$

For Grouped Data

Let $x_1, x_2, \ldots\ldots, x_n$ be a set of n observations occurring with frequencies $f_1, f_2, \ldots, f_n$ respectively, then mean deviation about mean is given by M.D.$(\bar{x}) = \frac{1}{N}\sum_{i=1}^{n} f_i|x_i - \bar{x}|$;

Mean deviation about median is given by M.D.(M)

$= \frac{1}{N}\sum_{i=1}^{n} f_i|x_i - M|$

Here, x_i are the mid-points of classes and $N = \sum_{i=1}^{n} f_i$ = Sum of frequencies

Shortcut Method

- **About mean :** M.D. $(\bar{x}) = \frac{1}{N}\cdot\sum_{i=1}^{n} f_i|x_i - \bar{x}|$, where mean, $\bar{x} = a + \frac{\sum_{i=1}^{n} f_i d_i}{N}\times h$, a is assumed mean, $d_i = \frac{x_i - a}{h}$ and h = size of interval.
- **About median :** M.D.$(M) = \frac{1}{N}\sum_{i=1}^{n} f_i|x_i - M|$, where median, $M = l + \frac{\frac{N}{2} - cf}{f}\times h$, N is sum of frequencies, l, f, h and cf are respectively the lower limit, the frequency of the median class, the width of the class interval and the cumulative frequency of the class just preceding the median class.

Analysis of Frequency Distributions

If the given data has mean $(\bar{x})$ and standard deviation (σ), then

Coefficient of variation (C.V.) $= \frac{\sigma}{\bar{x}}\times 100, \bar{x} \neq 0$

The data whose C.V. is less is said to be more consistent.

Variance and Standard Deviation

Mean of the squares of the deviations from mean is called variance and is denoted by σ^2.

The positive square root of variance is known as standard deviation. It is denoted by σ.

For ungrouped data	$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}$
For grouped data	$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{n} f_i\,(x_i - \bar{x})^2}$
Shortcut method	$\sigma = \frac{h}{N}\left[\sqrt{N\sum_{i=1}^{n} f_i\,u_i^2 - \left(\sum_{i=1}^{n} f_i\,u_i\right)^2}\right]$ where $u_i = \frac{x_i - a}{h}$, a = assumed mean, h = width of class-intervals

Properties of Standard Deviation

- S.D. is independent of change of origin.
- S.D. is not independent of change of scale.

Combined Variance of Two Series

If n_1, n_2 are the number of elements, $\bar{x}_1$, $\bar{x}_2$ are the means and σ_1, σ_2 are the standard deviations of two series respectively, then variance of combined series is

$$\sigma^2 = (\text{S.D.})^2 = \frac{n_1\left(\sigma_1^2 + d_1^2\right) + n_2\left(\sigma_2^2 + d_2^2\right)}{n_1 + n_2}$$

where $d_1 = \bar{x}_1 - \bar{x}$, $d_2 = \bar{x}_2 - \bar{x}$ and $\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

MATRICES

Class XII

CONCEPT MAP

MATRICES

A matrix is an ordered rectangular array of numbers or functions.
The numbers or functions are called the elements or entries of the matrix.

Order of a Matrix

A matrix having m rows and n columns is called a matrix of order $m \times n$.

Transpose of a Matrix

Transpose is obtained by interchanging rows and columns. If $A = [a_{ij}]_{m \times n}$, then A' or $A^T = [a_{ij}]_{n \times m}$

Properties

- $(A')' = A$
- $(AB)' = B'A'$
- $(A \pm B)' = A' \pm B'$
- $|A'| = |A|$

Operations		Properties
Addition and Subtraction	$A \pm B = C$ i.e., $[a_{ij}]_{m \times n} \pm [b_{ij}]_{m \times n}$ $= [a_{ij} \pm b_{ij}]_{m \times n}$ $= [c_{ij}]_{m \times n}$	• $A + B = B + A$ • $A + (B + C) = (A + B) + C$ • Additive inverse $= -A$ • Additive identity $= O$
Scalar Multiplication	$kA = B$ i.e., $[k\,a_{ij}] = [b_{ij}]$	• $k(A + B) = kA + kB$ • $(k + m)A = kA + mA$
Multiplication	$A_{m \times k} \times B_{k \times q} = C_{m \times q}$ i.e., $\left[\sum_{r=1}^{k} a_{ir}\, b_{rj}\right] = [c_{ij}]$	• If AB exists, then BA may or may not exist. • AB may or may not be equal to BA. • $(AB)\,C = A(BC)$ • $I_m \times A_{m \times n} = A_{m \times n} = A_{m \times n} \times I_n$ • $A(B + C) = AB + AC$; $(B + C)A = BA + CA$

Types of Matrices

Column Matrix : $A = [a_{ij}]_{m \times 1}$

Row Matrix : $A = [a_{ij}]_{1 \times n}$

Square Matrix : $A = [a_{ij}]_{m \times m}$

Diagonal Matix : $A = [a_{ij}]_{m \times m}$ where $a_{ij} = 0 \ \forall\ i \neq j$

Scalar Matrix : $A = [a_{ij}]_{n \times n}$ where $a_{ij} = \begin{cases} 0, \text{ if } i \neq j \\ k, \text{ if } i = j \end{cases}$ for some constant k

Zero Matrix : $A = [a_{ij}]$, where $a_{ij} = 0 \ \forall\ i = j$ and $i \neq j$

Identity Matrix : $A = [a_{ij}]$ where $a_{ij} = \begin{cases} 1 \text{ if } i = j \\ 0 \text{ if } i \neq j \end{cases}$

Special Matrices

Nilpotent Matrix : $A^k = O$ and $A^{k-1} \neq O$, $k \in Z^+$
Here, k is order of nilpotent matrix.

Involutory Matrix : $A^2 = I$

Orthogonal Matrix : $AA^T = A^T A = I$

Periodic Matrix : $A^k = A$, $k \in Z^+$

Idempotent Matrix : $A^2 = A$

Unitary Matrix : $AA^\theta = A^\theta A = I$

Symmetric Matrix : $A^T = A$

Skew Symmetric Matrix : $A^T = -A$
[where A is square matrix]

Inverse of a Matrix

If A and B are two square matrices such that $AB = BA = I$, then B is the inverse matrix of A and is denoted by A^{-1} and A is the inverse of B.

Properties

- A^{-1} is unique for any square matrix A.
- $(A^{-1})^{-1} = A$
- $(kA)^{-1} = A^{-1}/k$
- $(AB)^{-1} = B^{-1}A^{-1}$
- $(A^T)^{-1} = (A^{-1})^T$

CONCEPT BOOSTER

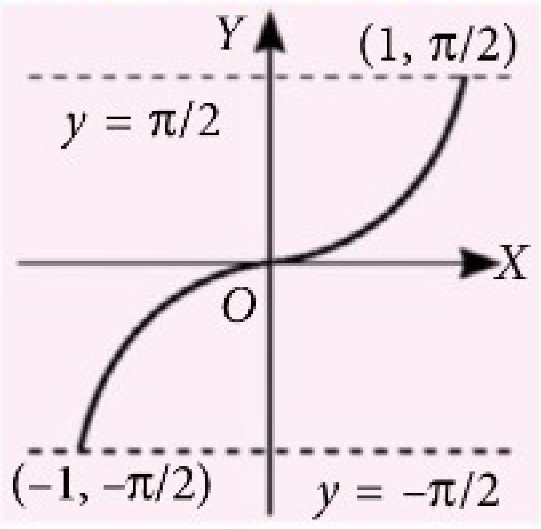
Inverse Trigonometric Functions

*ALOK KUMAR

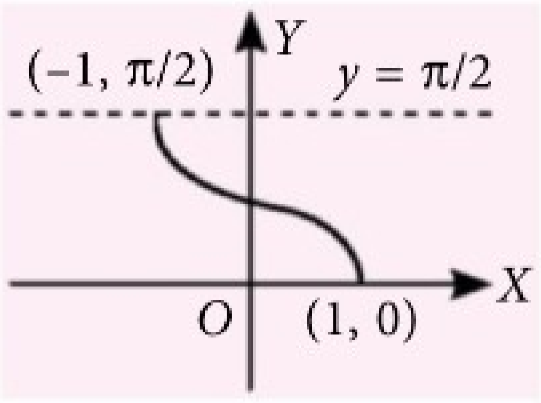
The inverse of a function $f : A \to B$ exists if f is one-one onto *i.e.*, a bijection and is given by $f(x) = y$ $\Rightarrow$ $f^{-1}(y) = x$.

GRAPHS OF INVERSE TRIGONOMETRIC FUNCTIONS

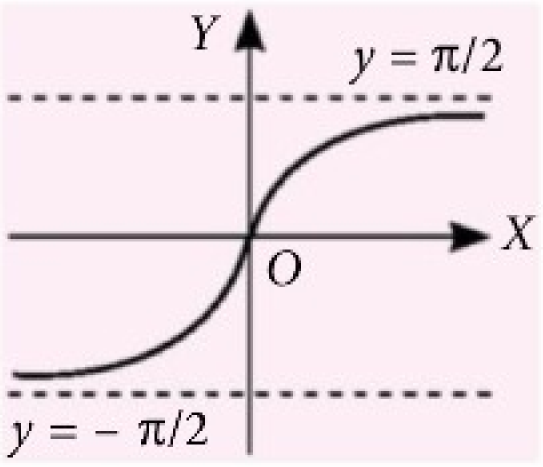
- Graph of $y = \sin^{-1}x$



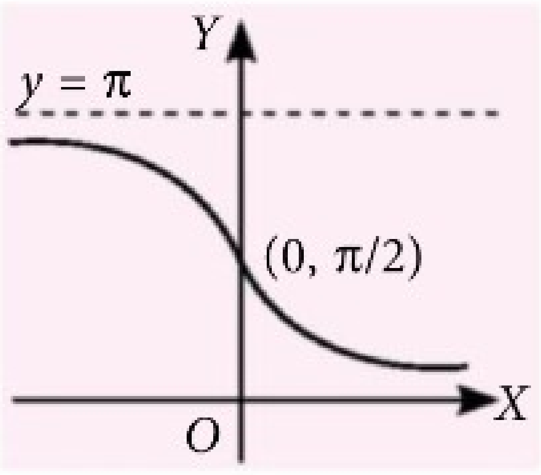
- Graph of $y = \cos^{-1}x$



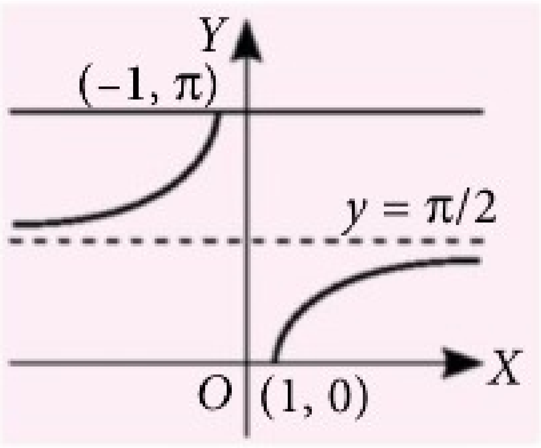
- Graph of $y = \tan^{-1}x$



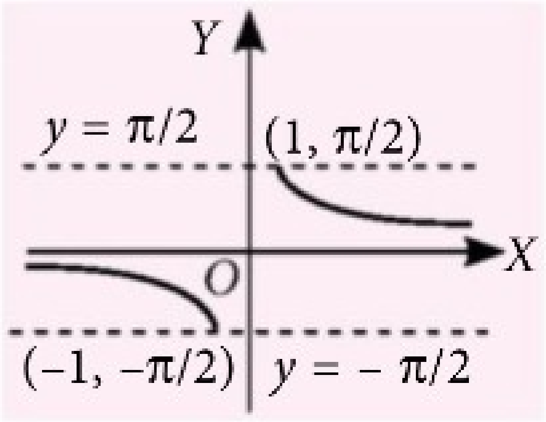
- Graph of $y = \cot^{-1}x$



- Graph of $y = \sec^{-1}x$



- Graph of $y = \text{cosec}^{-1}x$



DOMAIN AND RANGE OF INVERSE TRIGONOMETRIC FUNCTIONS

Functions	Domain	Range
$\sin^{-1}x$	$[-1, 1]$	$[-\pi/2, \pi/2]$
$\cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1}x$	$(-\infty, \infty)$	$(-\pi/2, \pi/2)$
$\cot^{-1}x$	$(-\infty, \infty)$	$(0, \pi)$
$\sec^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$[0, \pi/2) \cup (\pi/2, \pi]$
$\text{cosec}^{-1}x$	$(-\infty, -1] \cup [1,\infty)$	$[-\pi/2, 0) \cup (0, \pi/2]$

PRINCIPAL VALUES OF INVERSE TRIGONOMETRIC FUNCTIONS

Principal value for $x \geq 0$	Principal value for $x < 0$
$0 \leq \sin^{-1} x \leq \frac{\pi}{2}$	$-\frac{\pi}{2} \leq \sin^{-1} x < 0$
$0 \leq \cos^{-1} x \leq \frac{\pi}{2}$	$\frac{\pi}{2} < \cos^{-1} x \leq \pi$
$0 \leq \tan^{-1} x < \frac{\pi}{2}$	$-\frac{\pi}{2} < \tan^{-1} x < 0$
$0 < \cot^{-1} x \leq \frac{\pi}{2}$	$\frac{\pi}{2} < \cot^{-1} x < \pi$
$0 \leq \sec^{-1} x < \frac{\pi}{2}$	$\frac{\pi}{2} < \sec^{-1} x \leq \pi$
$0 < \text{cosec}^{-1} x \leq \frac{\pi}{2}$	$-\frac{\pi}{2} \leq \text{cosec}^{-1} x < 0$

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PROPERTIES OF INVERSE TRIGONOMETRIC FUNCTIONS

- $\sin^{-1}(\sin x) = x,\ \forall\ -\frac{\pi}{2} \le x \le \frac{\pi}{2}$
- $\cos^{-1}(\cos x) = x,\ \forall\ 0 \le x \le \pi$
- $\tan^{-1}(\tan x) = x\ ,\ \forall -\frac{\pi}{2} < x < \frac{\pi}{2}$
- $\cot^{-1}(\cot x) = x,\ \forall\ 0 < x < \pi$
- $\sec^{-1}(\sec x) = x\ ,\ \forall\ x \in\ [0, \pi] - \left\{\frac{\pi}{2}\right\}$
- $\text{cosec}^{-1}(\text{cosec}\ x) = x,\ \forall\ x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
- $\sin(\sin^{-1}x) = x,\ \forall\ -1 \le x \le 1$
- $\cos(\cos^{-1}x) = x,\ \forall\ -1 \le x \le 1$
- $\tan(\tan^{-1}x) = x,\ \forall\ -\infty < x < \infty$
- $\cot(\cot^{-1}x) = x,\ \forall\ -\infty < x < \infty$
- $\sec(\sec^{-1}x) = x\ ,\ \forall\ x \in\ (-\infty, -1] \cup [1, \infty)$
- $\text{cosec}(\text{cosec}^{-1}x) = x\ ,\ \forall\ x \in\ (-\infty, -1] \cup [1, \infty)$
- $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\ ,\ \forall\ -1 \le x \le 1$
- $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}\ ,\ \forall\ x \in\ R$
- $\sec^{-1}x + \text{cosec}^{-1}x = \frac{\pi}{2}\ ,\ \ \forall\ x \le -1 \text{ or } x \ge 1$
- $\sin^{-1}(-x) = -\sin^{-1}x\ \ ,\ \forall\ -1 \le x \le 1$
- $\cos^{-1}(-x) = \pi - \cos^{-1}x\ ,\ \forall -1 \le x \le 1$
- $\tan^{-1}(-x) = -\tan^{-1}x,\ \forall\ -\infty < x < \infty$
- $\cot^{-1}(-x) = \pi - \cot^{-1}x,\ \forall\ -\infty < x < \infty$
- $\sec^{-1}(-x) = \pi - \sec^{-1}x\ ,\ \forall\ |x| \ge 1$
- $\text{cosec}^{-1}(-x) = -\ \text{cosec}^{-1}x,\ \forall\ |x| \ge 1$
- $\sin^{-1}x = \cos^{-1}\sqrt{1-x^2}$
  $$= \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) = \cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$$
  $$= \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right) = \text{cosec}^{-1}\left(\frac{1}{x}\right)$$
- $\cos^{-1}x = \sin^{-1}\sqrt{1-x^2}$
  $$= \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) = \sec^{-1}\left(\frac{1}{x}\right)$$
  $$= \text{cosec}^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right) = \cot^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$
- $\tan^{-1}x = \sec^{-1}\sqrt{1+x^2} = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$
  $$= \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) = \text{cosec}^{-1}\left(\frac{\sqrt{1+x^2}}{x}\right) = \cot^{-1}\left(\frac{1}{x}\right)$$
- $\sin^{-1}\left(\frac{1}{x}\right) = \text{cosec}^{-1}x\ ,\ \forall\ x \in\ (-\infty, -1] \cup [1, \infty)$
- $\cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1}x,\ \forall\ x \in (-\infty,\ -1] \cup [1, \infty)$
- $\tan^{-1}\left(\frac{1}{x}\right) = \begin{cases} \cot^{-1}x, & \text{for } x > 0 \\ -\pi + \cot^{-1}x, & \text{for } x < 0 \end{cases}$
- $\tan^{-1}x + \tan^{-1}y$
  $$= \begin{cases} \tan^{-1}\left(\frac{x+y}{1-xy}\right), & \forall\ x > 0,\ y > 0 \text{ and } xy < 1 \\ \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right), & \forall\ x > 0,\ y > 0 \text{ and } xy > 1 \\ -\pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right), & \forall\ x < 0,\ y < 0 \text{ and } xy > 1 \end{cases}$$
- $\tan^{-1}x - \tan^{-1}y$
  $$= \begin{cases} \tan^{-1}\left(\frac{x-y}{1+xy}\right), & \forall\ xy > -1 \\ \pi + \tan^{-1}\left(\frac{x-y}{1+xy}\right), & \forall\ x > 0,\ y < 0 \text{ and } xy < -1 \\ -\pi + \tan^{-1}\left(\frac{x-y}{1+xy}\right), & \forall\ x < 0,\ y > 0 \text{ and } xy < -1 \end{cases}$$
- $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right),$
  $\forall x,\ y,\ z \ge 0$
- If $x_1, x_2, x_3, \ldots, x_n \in R$, then $\tan^{-1}x_1 + \tan^{-1}x_2 + \ldots + \tan^{-1}x_n = \tan^{-1}\left(\frac{S_1 - S_3 + S_5 - S_7 + \ldots}{1 - S_2 + S_4 - S_6 + \ldots}\right)$
  where S_k denotes the sum of the products of $x_1, x_2, \ldots, x_n$ taken k at a time.
- $\cot^{-1}x + \cot^{-1}y = \cot^{-1}\left(\frac{xy-1}{y+x}\right)$
- $\cot^{-1}x - \cot^{-1}y = \cot^{-1}\left(\frac{xy+1}{y-x}\right)$
- $\sin^{-1}x + \sin^{-1}y$
  $$= \begin{cases} \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2}), \forall\ x \ge 0,\ y \ge 0,\ x^2 + y^2 \le 1 \\ \pi - \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2}), \forall\ x \ge 0,\ y \ge 0,\ x^2 + y^2 > 1 \\ -\pi - \sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\},\ \forall -1 \le x,\ y < 0,\ x^2 + y^2 > 1 \end{cases}$$

- $\sin^{-1}x - \sin^{-1}y$

$$= \begin{cases} \sin^{-1}(x\sqrt{1-y^2} - y\sqrt{1-x^2}), \forall\, x \ge 0,\, y \ge 0, \\ \qquad\qquad x^2 + y^2 \le 1 \\ \pi - \sin^{-1}(x\sqrt{1-y^2} - y\sqrt{1-x^2}), \forall\, x \ge 0,\, y \ge 0, \\ \qquad\qquad x^2 + y^2 > 1 \\ -\pi - \sin^{-1}\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\}, \\ \qquad \forall -1 \le x < 0,\, 0 < y \le 1,\, x^2 + y^2 > 1 \end{cases}$$

- $\cos^{-1}x + \cos^{-1}y$

$$= \begin{cases} \cos^{-1}(xy - \sqrt{1-x^2}\sqrt{1-y^2}), \\ \qquad\qquad \forall\, x \ge 0,\, y \ge 0,\, x^2 + y^2 \le 1 \\ \pi - \cos^{-1}(xy - \sqrt{1-x^2}\sqrt{1-y^2}), \\ \qquad\qquad \forall\, x \ge 0,\, y \ge 0,\, x^2 + y^2 > 1 \end{cases}$$

- $\cos^{-1}x - \cos^{-1}y$

$$= \begin{cases} \cos^{-1}(xy + \sqrt{1-x^2}\sqrt{1-y^2}), \forall\, x \ge 0,\, y \ge 0,\, y > x \\ -\cos^{-1}(xy + \sqrt{1-x^2}\sqrt{1-y^2}), \forall\, x \ge 0,\, y \ge 0,\, y < x \end{cases}$$

INVERSE TRIGONOMETRIC RATIOS OF MULTIPLE ANGLES

- $$2\sin^{-1}x = \begin{cases} \sin^{-1}(2x\sqrt{1-x^2}), \quad \forall -\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}} \\ \pi - \sin^{-1}(2x\sqrt{1-x^2}), \forall\, \frac{1}{\sqrt{2}} < x \le 1 \\ -\pi - \sin^{-1}(2x\sqrt{1-x^2}), \forall -1 \le x < \frac{-1}{\sqrt{2}} \end{cases}$$

- $$3\sin^{-1}x = \begin{cases} \sin^{-1}(3x - 4x^3), \forall -\frac{1}{2} \le x \le \frac{1}{2} \\ \pi - \sin^{-1}(3x - 4x^3), \forall\, \frac{1}{2} < x \le 1 \\ -\pi - \sin^{-1}(3x - 4x^3), \forall -1 \le x < -\frac{1}{2} \end{cases}$$

- $$2\cos^{-1}x = \begin{cases} \cos^{-1}(2x^2 - 1), \forall\, 0 \le x \le 1 \\ 2\pi - \cos^{-1}(2x^2 - 1), \forall -1 \le x \le 0 \end{cases}$$

- $$3\cos^{-1}x = \begin{cases} \cos^{-1}(4x^3 - 3x), \forall\, \frac{1}{2} \le x \le 1 \\ 2\pi - \cos^{-1}(4x^3 - 3x), \forall -\frac{1}{2} \le x \le \frac{1}{2} \\ 2\pi + \cos^{-1}(4x^3 - 3x), \forall -1 \le x \le -\frac{1}{2} \end{cases}$$

- $$2\tan^{-1}x = \begin{cases} \sin^{-1}\left(\frac{2x}{1+x^2}\right), \forall\, -1 \le x \le 1 \\ -\pi - \sin^{-1}\left(\frac{2x}{1+x^2}\right), \forall\, x < -1 \\ \pi - \sin^{-1}\left(\frac{2x}{1+x^2}\right), \forall\, x > 1 \\ \pi + \tan^{-1}\frac{2x}{1-x^2}, \forall x > 1 \\ -\pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right), \forall\, x < -1 \\ \tan^{-1}\frac{2x}{1-x^2}, \forall\, |x| < 1 \\ \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), \forall\, 0 \le x < \infty \\ -\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), \forall -\infty < x \le 0 \end{cases}$$

- $$3\tan^{-1}x = \begin{cases} \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right), \forall -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}} \\ \pi + \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right), \forall\, x > \frac{1}{\sqrt{3}} \\ -\pi + \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right), \forall\, x < -\frac{1}{\sqrt{3}} \end{cases}$$

PROBLEMS

Single Correct Answer Type

1. $\tan[\sec^{-1}\sqrt{1+x^2}] =$

(a) $\frac{1}{x}$ (b) x

(c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{1+x^2}}$

2. $\tan^{-1}\left[\frac{\cos x}{1 + \sin x}\right] =$

(a) $\frac{\pi}{4} - \frac{x}{2}$ (b) $\frac{\pi}{4} + \frac{x}{2}$

(c) $\frac{x}{2}$ (d) $\frac{\pi}{4} - x$

3. $\tan^{-1}\dfrac{1}{\sqrt{x^2-1}}=$

(a) $\dfrac{\pi}{2}+\operatorname{cosec}^{-1}x$ (b) $\dfrac{\pi}{2}+\sec^{-1}x$

(c) $\operatorname{cosec}^{-1}x$ (d) $\sec^{-1}x$

4. $\sec^2(\tan^{-1}2)+\operatorname{cosec}^2(\cot^{-1}3)=$

(a) 5 (b) 13 (c) 15 (d) 6

5. If $\sin\left(\sin^{-1}\dfrac{1}{5}+\cos^{-1}x\right)=1$, then x is equal to

(a) 1 (b) 0 (c) $\dfrac{4}{5}$ (d) $\dfrac{1}{5}$

6. If $\sin^{-1}x=\theta+\beta$ and $\sin^{-1}y=\theta-\beta$, then $1+xy=$

(a) $\sin^2\theta+\sin^2\beta$ (b) $\sin^2\theta+\cos^2\beta$

(c) $\cos^2\theta+\cos^2\beta$ (d) $\cos^2\theta+\sin^2\beta$

7. If $\sin^{-1}\dfrac{1}{3}+\sin^{-1}\dfrac{2}{3}=\sin^{-1}x$, then x is equal to

(a) 0 (b) $\dfrac{\sqrt{5}-4\sqrt{2}}{9}$

(c) $\dfrac{\sqrt{5}+4\sqrt{2}}{9}$ (d) $\dfrac{\pi}{2}$

8. $\tan(\cos^{-1}x)$ is equal to

(a) $\dfrac{\sqrt{1-x^2}}{x}$ (b) $\dfrac{x}{1+x^2}$

(c) $\dfrac{\sqrt{1+x^2}}{x}$ (d) $\sqrt{1-x^2}$

9. The smallest and the largest values of $\tan^{-1}\left(\dfrac{1-x}{1+x}\right)$, $0\le x\le 1$ are

(a) $0,\pi$ (b) $0,\dfrac{\pi}{4}$ (c) $-\dfrac{\pi}{4},\dfrac{\pi}{4}$ (d) $\dfrac{\pi}{4},\dfrac{\pi}{2}$

10. The value of x which satisfies the equation $\tan^{-1}x=\sin^{-1}\left(\dfrac{3}{\sqrt{10}}\right)$ is

(a) 3 (b) -3 (c) $\dfrac{1}{3}$ (d) $-\dfrac{1}{3}$

11. If $\theta=\sin^{-1}[\sin(-600°)]$, then one of the possible value of θ is

(a) $\dfrac{\pi}{3}$ (b) $\dfrac{\pi}{2}$ (c) $\dfrac{2\pi}{3}$ (d) $\dfrac{-2\pi}{3}$

12. $\sin[\cot^{-1}(\cos\tan^{-1}x)]=$

(a) $\dfrac{x}{\sqrt{x^2+2}}$ (b) $\dfrac{x}{\sqrt{x^2+1}}$

(c) $\dfrac{1}{\sqrt{x^2+2}}$ (d) $\sqrt{\dfrac{x^2+1}{x^2+2}}$

13. If $\sin(\cot^{-1}(x+1))=\cos(\tan^{-1}x)$, then $x=$

(a) $-\dfrac{1}{2}$ (b) $\dfrac{1}{2}$ (c) 0 (d) $\dfrac{9}{4}$

14. $2\tan^{-1}\dfrac{1}{3}+\tan^{-1}\dfrac{1}{2}=$

(a) 90° (b) 60° (c) 45° (d) $\tan^{-1}2$

15. $\tan\left[\cos^{-1}\dfrac{4}{5}+\tan^{-1}\dfrac{2}{3}\right]=$

(a) 6/17 (b) 17/6 (c) 7/16 (d) 16/7

16. If $\cot^{-1}\dfrac{3}{4}+\sin^{-1}\dfrac{5}{13}=\sin^{-1}(k)$, then $k=$

(a) $\dfrac{63}{65}$ (b) $\dfrac{12}{13}$ (c) $\dfrac{65}{68}$ (d) $\dfrac{5}{12}$

17. If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$, then

(a) $x^2+y^2+z^2+xyz=0$

(b) $x^2+y^2+z^2+2xyz=0$

(c) $x^2+y^2+z^2+xyz=1$

(d) $x^2+y^2+z^2+2xyz=1$

18. $\cos\left[2\cos^{-1}\dfrac{1}{5}+\sin^{-1}\dfrac{1}{5}\right]=$

(a) $\dfrac{2\sqrt{6}}{5}$ (b) $-\dfrac{2\sqrt{6}}{5}$ (c) $\dfrac{1}{5}$ (d) $-\dfrac{1}{5}$

19. $\cot^{-1}\dfrac{xy+1}{x-y}+\cot^{-1}\dfrac{yz+1}{y-z}+\cot^{-1}\dfrac{zx+1}{z-x}=$

(a) 0 (b) 1

(c) $\cot^{-1}x+\cot^{-1}y+\cot^{-1}z$

(d) None of these

20. If $\tan^{-1}\dfrac{a+x}{a}+\tan^{-1}\dfrac{a-x}{a}=\dfrac{\pi}{6}$, then $x^2=$

(a) $2\sqrt{3}a$ (b) $\sqrt{3}a$ (c) $2\sqrt{3}a^2$ (d) $\sqrt{3}a^2$

21. $\tan^{-1}\dfrac{1-x^2}{2x}+\cos^{-1}\dfrac{1-x^2}{1+x^2}=$

Monthly Test Drive – 6 CLASS XI ANSWER KEY

1.	(d)	**2.**	(b)	**3.**	(d)	**4.**	(c)
5.	(d)	**6.**	(d)	**7.**	(a, b, c)	**8.**	(a)
9.	(b, c, d)	**10.**	(a, c)	**11.**	(a, b, c)	**12.**	(a,b,c,d)
13.	(a, b, c, d)	**14.**	(c)	**15.**	(a)	**16.**	(d)
17.	(2007)	**18.**	(1080)	**19.**	(4)	**20.**	(4)

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) π (d) 0

22. If $\sin^{-1}\left(\frac{2a}{1+a^2}\right)+\sin^{-1}\left(\frac{2b}{1+b^2}\right)=2\tan^{-1}x$, then $x=$

(a) $\frac{a-b}{1+ab}$ (b) $\frac{b}{1+ab}$

(c) $\frac{b}{1-ab}$ (d) $\frac{a+b}{1-ab}$

23. $4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{70}+\tan^{-1}\frac{1}{99}=$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

24. If $\sin^{-1}x+\sin^{-1}y=\frac{2\pi}{3}$, then $\cos^{-1}x + \cos^{-1}y =$

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) π

25. $\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{x-y}{x+y}\right)$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $-\frac{3\pi}{4}$

26. If $\sin^{-1}\frac{x}{5}+\text{cosec}^{-1}\left(\frac{5}{4}\right)=\frac{\pi}{2}$, then $x =$

(a) 4 (b) 5 (c) 1 (d) 3

27. If $x^2 + y^2 + z^2 = r^2$, then

$$\tan^{-1}\left(\frac{xy}{zr}\right)+\tan^{-1}\left(\frac{yz}{xr}\right)+\tan\left(\frac{zx}{yr}\right)=$$

(a) π (b) $\frac{\pi}{2}$ (c) 0 (d) $\frac{\pi}{4}$

28. If $a<\frac{1}{32}$, then the number of solution of $(\sin^{-1}x)^3 + (\cos^{-1}x)^3 = a\pi^3$ is

(a) 0 (b) 1 (c) 2 (d) Infinite

29. If $\tan(x + y) = 33$ and $x = \tan^{-1}3$, then y will be

(a) 0.3 (b) $\tan^{-1}(1.3)$

(c) $\tan^{-1}(0.3)$ (d) $\tan^{-1}\left(\frac{1}{18}\right)$

30. $\tan^{-1}\left(\frac{1}{11}\right)+\tan^{-1}\left(\frac{2}{12}\right)=$

(a) $\tan^{-1}\left(\frac{33}{132}\right)$ (b) $\tan^{-1}\left(\frac{1}{2}\right)$

(c) $\tan^{-1}\left(\frac{132}{33}\right)$ (d) $\tan^{-1}\left(\frac{17}{65}\right)$

31. If $\tan^{-1}x+2\cot^{-1}x=\frac{2\pi}{3}$, then $x =$

(a) $\sqrt{2}$ (b) 3 (c) $\sqrt{3}$ (d) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

32. If $4\sin^{-1}x + \cos^{-1}x = \pi$, then x is equal to

(a) 0 (b) $\frac{1}{2}$ (c) $-\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{2}}$

Multiple Correct Answer Type

33. $2\cot^{-1}7+\cos^{-1}\frac{3}{5}$ is equal to

(a) $\cot^{-1}\left(\frac{44}{117}\right)$ (b) $\text{cosec}^{-1}\left(\frac{125}{117}\right)$

(c) $\tan^{-1}\left(\frac{4}{117}\right)$ (d) $\cos^{-1}\left(\frac{44}{125}\right)$

34. If $0<x<1$, then $\tan^{-1}\frac{\sqrt{1-x^2}}{1+x}$ is equal to

(a) $\frac{1}{2}\cos^{-1}x$ (b) $\cos^{-1}\sqrt{\frac{1+x}{2}}$

(c) $\sin^{-1}\sqrt{\frac{1-x}{2}}$ (d) None of these

35. If the equation $\sin^{-1}(x^2+x+1)+\cos^{-1}(\lambda x+1)=\pi/2$ has exactly two distinct solutions, then value(s) of λ can be

(a) 0 (b) 1

(c) -1 (d) 2

36. If α is a real number for which $f(x) = \ln \cos^{-1}x$ is defined, then a possible value of $[\alpha]$ is (where [.] denotes greatest integer function)

(a) 0 (b) 1

(c) -1 (d) -2

37. If $f(x) = \sin^{-1}x + \cos^{-1}x$, then $\pi/2$ is equal to

(a) $f\left(-\frac{1}{2}\right)$ (b) $f(k^2 - 2k + 3)$, $k \in R$

(c) $f\left(\frac{1}{1+k^2}\right)$, $k\in R$ (d) $f(-2)$

Assertion & Reason Type

Directions : *In the following questions, Statement-1 is followed by Statement-2. Mark the correct choice as :*

(a) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is true, Statement–2 is true, Statement-2 is not a correct explanation for Statement-1

(c) Statement-1 is true, Statement-2 is false

(d) Statement-1 is false, Statement-2 is true

38. **Statement-1 :** The number of solutions of the equation $\sin x + \cos x = \sin^{-1}x + \cos^{-1}x$ is zero.

Statement-2 : $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\ \forall x \in [-1, 1]$ and the maximum value of $(\sin x + \cos x)$ is $\sqrt{2}$ for $x \in R$.

39. **Statement-1 :** If $\sum_{i=1}^{2n} \sin^{-1} x_i = n\pi, n \in N$, then $\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i^3$

Statement-2 : $-\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}, \forall x \in [-1, 1]$

40. **Statement-1 :** If $\alpha \in (-\pi/2, 0)$, then the value of $2\tan^{-1}(\operatorname{cosec}\alpha) + \tan^{-1}(2\sin\alpha \sec^2\alpha)$ is $-\pi$.

Statement-2 : If $x < 0$, then $\tan^{-1}x + \tan^{-1}\frac{1}{x} = -\frac{\pi}{2}$

41. **Statement-1 :** $\sec^{-1} 5 < \tan^{-1} 7$

Statement-2 : $\sec^{-1}x < \tan^{-1}x$, if $x \ge 1$ and $\sec^{-1}x > \tan^{-1}x$ if $x \le -1$ and $\tan^{-1}x_1 > \tan^{-1}x_2$ if $x_1 > x_2$.

42. **Statement-1 :** $\sin^{-1}\left(\frac{1}{\sqrt{e}}\right) > \tan^{-1}\left(\frac{1}{\sqrt{\pi}}\right)$

Statement-2 : $\sin^{-1}x > \tan^{-1}y$ for $x > y$, $\forall\ x, y \in (0, 1)$.

Comprehension Type

Paragraph for Question No. 43 to 45

It is given that $A = (\tan^{-1}x)^3 + (\cot^{-1}x)^3$ where $x > 0$ and $B = (\cos^{-1}t)^2 + (\sin^{-1}t)^2$, where $t \in \left[0, \frac{1}{\sqrt{2}}\right]$ and $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ for $-1 \le x \le 1$ and $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$ for all $x \in R$.

43. The interval in which A lies is

(a) $\left[\frac{\pi^3}{7}, \frac{\pi^3}{2}\right)$ (b) $\left[\frac{\pi^3}{32}, \frac{\pi^3}{8}\right)$

(c) $\left[\frac{\pi^3}{40}, \frac{\pi^3}{10}\right)$ (d) None of these

44. The maximum value of B is

(a) $\frac{\pi^2}{8}$ (b) $\frac{\pi^2}{16}$ (c) $\frac{\pi^2}{4}$ (d) $\frac{\pi^2}{6}$

45. If least value of A is λ and maximum value of B is μ, then $\cot^{-1}\cot\left(\frac{\lambda - \mu\pi}{\mu}\right) =$

(a) $\frac{\pi}{8}$ (b) $-\frac{\pi}{8}$ (c) $\frac{7\pi}{8}$ (d) $-\frac{7\pi}{8}$

Matrix-Match Type

46. If [.] represents greatest integer function, then match the following.

Column-I		Column-II	
A.	If $f(x) = \sin^{-1} x$ and $\lim_{x \to (1/2)^+} f(3x - 4x^3) = a - 3 \lim_{x \to (1/2)^+} f(x)$, then $[a] =$	p.	2
B.	If $f(x) = \tan^{-1} g(x)$, where $g(x) = \frac{3x - x^3}{1 - 3x^2}$ and $\lim_{h \to 0} \frac{f(a+3h) - f(a)}{3h} = \frac{3}{1+a^2}$, when $\frac{-1}{3} < a < \frac{1}{\sqrt{3}}$, then $\left[\lim_{h \to 0} \frac{f\left(\frac{1}{2} + 6h\right) - f\left(\frac{1}{2}\right)}{6h}\right] =$	q.	3
C.	If $\cos^{-1}(4x^3 - 3x) = a + b\cos^{-1}x$ for $-1 < x < \frac{-1}{2}$, then $[a + b + 2] =$	r.	–3
D.	If $f(x) = \cos^{-1}(4x^3 - 3x)$ and $\lim_{x \to (1/2)^+} f'(x) = a$ and $\lim_{x \to (1/2)^-} f'(x) = b$, then $[a + b - 3] =$	s.	–2

Numerical Value Type

47. The number of ordered pairs (x, y) satisfying the system of equations $(\cos^{-1}x)^2 + \sin^{-1}y = 1$ and $\cos^{-1}x + (\sin^{-1}y)^2 = 1$ is (are)

48. If $S = \sum_{r=1}^{50} \tan^{-1}\left(\frac{2r}{2 + r^2 + r^4}\right) = \tan^{-1}\left(\frac{\lambda}{\mu}\right)$, where $\left(\frac{\lambda}{\mu}\right)$ is in simplest form, then $\mu - \lambda =$

49. Let $P = \tan^{-1}\left[\frac{3\sin 2\alpha}{5 + 3\cos 2\alpha}\right] + \tan^{-1}\left[\frac{\tan\alpha}{4}\right]$, where $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$, then $\frac{P}{\alpha}$ equals

50. If $(\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$, then $|x|$ equals

SOLUTIONS

1. **(b) :** $\tan\left(\sec^{-1}\sqrt{1+x^2}\right) = \tan\left(\sec^{-1}\sqrt{1+\tan^2\theta}\right)$

(Putting $x = \tan\theta$)

$= \tan(\sec^{-1}(\sec\theta)) = \tan\theta = x$

2. (a) : $\tan^{-1}\left[\frac{\cos x}{1+\sin x}\right]=\tan^{-1}\left[\frac{\sin(\pi/2-x)}{1+\cos(\pi/2-x)}\right]$

$$=\tan^{-1}\left[\frac{2\sin(\pi/4-x/2)\cos(\pi/4-x/2)}{2\cos^2(\pi/4-x/2)}\right]$$

$$=\tan^{-1}\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)=\frac{\pi}{4}-\frac{x}{2}$$

3. (c) : $\tan^{-1}\frac{1}{\sqrt{x^2-1}}=\tan^{-1}\frac{1}{\sqrt{\operatorname{cosec}^2\theta-1}}$
(Putting $x = \operatorname{cosec}\theta$)

$$=\tan^{-1}\left(\frac{1}{\cot\theta}\right)=\theta=\operatorname{cosec}^{-1}x$$

4. (c) : Let $\tan^{-1} 2 = \alpha \Rightarrow \tan\alpha = 2$
and $\cot^{-1} 3 = \beta \Rightarrow \cot\beta = 3$
Now, $\sec^2(\tan^{-1} 2) + \operatorname{cosec}^2(\cot^{-1} 3)$
$= \sec^2\alpha + \operatorname{cosec}^2\beta = 1 + \tan^2\alpha + 1 + \cot^2\beta$
$= 2 + (2)^2 + (3)^2 = 15$

5. (d) : We have, $\sin^{-1}\frac{1}{5}+\cos^{-1}x=\frac{\pi}{2}$

$$\Rightarrow \quad \sin^{-1}\frac{1}{5}=\frac{\pi}{2}-\cos^{-1}x=\sin^{-1}x \quad \Rightarrow \quad x=\frac{1}{5}$$

6. (b) : Obviously $x = \sin(\theta + \beta)$ and $y = \sin(\theta - \beta)$
$\therefore \; 1 + xy = 1 + \sin(\theta + \beta)\sin(\theta - \beta)$
$= 1 + \sin^2\theta - \sin^2\beta = \sin^2\theta + \cos^2\beta$

7. (c) : We have, $\sin^{-1}x=\sin^{-1}\frac{1}{3}+\sin^{-1}\frac{2}{3}$

$$=\sin^{-1}\left[\frac{1}{3}\sqrt{1-\frac{4}{9}}+\frac{2}{3}\sqrt{1-\frac{1}{9}}\right]=\sin^{-1}\left[\frac{\sqrt{5}+4\sqrt{2}}{9}\right]$$

8. (a) : Let $\cos^{-1}x = \theta$. Then $x = \cos\theta$

$$\Rightarrow \quad \tan\theta=\sqrt{\sec^2\theta-1}=\sqrt{\frac{1}{x^2}-1}=\frac{\sqrt{1-x^2}}{x}$$

$$\therefore \quad \tan(\cos^{-1}x)=\tan\theta=\frac{\sqrt{1-x^2}}{x}$$

9. (b) : We have, $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{\pi}{4}-\tan^{-1}x$

Since $0\le x\le 1 \Rightarrow 0\le \tan^{-1}x\le\frac{\pi}{4}$

$$\Rightarrow \quad 0\ge -\tan^{-1}x\ge\frac{-\pi}{4}\Rightarrow\frac{\pi}{4}\ge\frac{\pi}{4}-\tan^{-1}x\ge 0$$

$$\Rightarrow \quad \frac{\pi}{4}\ge\tan^{-1}\left(\frac{1-x}{1+x}\right)\ge 0$$

10. (a) : Given, $\tan^{-1}x=\sin^{-1}\left[\frac{3}{\sqrt{10}}\right]$

$$\Rightarrow \quad x=\tan\left\{\sin^{-1}\left[\frac{3}{\sqrt{10}}\right]\right\}=\tan\,\{\tan^{-1}3\} \;\Rightarrow\; x=3$$

11. (a) : $\theta = \sin^{-1}[\sin(-600°)]$
$\Rightarrow \; \theta = \sin^{-1}[-(\sin 240°)] = \sin^{-1}[-\sin(180° + 60°)]$

$$\Rightarrow \quad \theta=\sin^{-1}(\sin 60°)=\sin^{-1}\left[\sin\left(\frac{\pi}{3}\right)\right]=\frac{\pi}{3}$$

12. (d)

13. (a) : $\sin[\cot^{-1}(x+1)]=\sin\left(\sin^{-1}\frac{1}{\sqrt{x^2+2x+2}}\right)$

$$=\frac{1}{\sqrt{x^2+2x+2}} \qquad \text{...(i)}$$

and $\cos(\tan^{-1}x)=\cos\left(\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right)=\frac{1}{\sqrt{1+x^2}}$ ...(ii)

From (i) and (ii), $x=-\frac{1}{2}$

14. (d) : $2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{2}=\tan^{-1}\left(\frac{\frac{2}{3}}{1-\frac{1}{9}}\right)+\tan^{-1}\left(\frac{1}{2}\right)$

$$=\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{3}{4}=\tan^{-1}\left(\frac{\frac{1}{2}+\frac{3}{4}}{1-\frac{1}{2}\times\frac{3}{4}}\right)=\tan^{-1}(2)$$

15. (b) : $\tan\left[\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right]$

$$=\tan\left[\tan^{-1}\frac{\sqrt{\left(1-\frac{16}{25}\right)}}{\frac{4}{5}}+\tan^{-1}\frac{2}{3}\right]$$

$$=\tan\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{2}{3}\right)=\tan\left(\tan^{-1}\frac{17}{6}\right)=\frac{17}{6}$$

16. (a) : Let $\cot^{-1}\frac{3}{4}=\theta\Rightarrow\cot\theta=\frac{3}{4}$

and $\sin\theta=\frac{1}{\sqrt{1+\cot^2\theta}}=\frac{1}{\sqrt{1+(9/16)}}=\frac{4}{5}$

Hence, $\cot^{-1}\frac{3}{4}+\sin^{-1}\frac{5}{13}=\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{5}{13}$

$$=\sin^{-1}\left[\frac{4}{5}.\sqrt{1-\frac{25}{169}}+\frac{5}{13}.\sqrt{1-\frac{16}{25}}\right]=\sin^{-1}\frac{63}{65}$$

17. (d) : Given that $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$
$\Rightarrow \; \cos^{-1}(x) + \cos^{-1}(y) = \cos^{-1}(-1) = \cos^{-1}(z)$

$$\Rightarrow \quad \cos^{-1}(xy-\sqrt{1-x^2}\sqrt{1-y^2})-\cos^{-1}\{(-1)(z)\}$$

$$\Rightarrow \quad (xy+z)=\sqrt{(1-x^2)\,(1-y^2)}$$

Squaring both sides, we get $x^2 + y^2 + z^2 + 2xyz = 1$.

Put $x=y=z=\frac{1}{2}$, so that $\cos^{-1}\frac{1}{2}+\cos^{-1}\frac{1}{2}+\cos^{-1}\frac{1}{2}=\pi$

Obviously option (d) holds for these values of x, y and z.

18. (b) : $\cos\left(\cos^{-1}\frac{1}{5}+\sin^{-1}\frac{1}{5}+\cos^{-1}\frac{1}{5}\right)$

$=\cos\left(\frac{\pi}{2}+\cos^{-1}\frac{1}{5}\right)$

$=-\sin\left(\cos^{-1}\frac{1}{5}\right)=-\sin\left(\sin^{-1}\sqrt{\frac{24}{25}}\right)=-\frac{2\sqrt{6}}{5}$

19. (a) : $\cot^{-1}\frac{xy+1}{x-y}+\cot^{-1}\frac{yz+1}{y-z}+\cot^{-1}\frac{zx+1}{z-x}$

$=\cot^{-1}y-\cot^{-1}x+\cot^{-1}z-\cot^{-1}y+\cot^{-1}x-\cot^{-1}z=0$

20. (c) : We have, $\tan^{-1}\frac{a+x}{a}+\tan^{-1}\frac{a-x}{a}=\frac{\pi}{6}$

$\Rightarrow\ \tan^{-1}\left(\frac{\frac{a+x}{a}+\frac{a-x}{a}}{1-\frac{a+x}{a}\cdot\frac{a-x}{a}}\right)=\frac{\pi}{6}$

$\Rightarrow\ \frac{2a^2}{x^2}=\tan\frac{\pi}{6}=\frac{1}{\sqrt{3}}\Rightarrow x^2=2\sqrt{3}a^2$

21. (b) : $\tan^{-1}\frac{1-x^2}{2x}+\cos^{-1}\frac{1-x^2}{1+x^2}$

$=\tan^{-1}\left(\frac{1-\tan^2\theta}{2\tan\theta}\right)+\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$=\tan^{-1}(\cot 2\theta)+\cos^{-1}(\cos 2\theta)=\frac{\pi}{2}-2\theta+2\theta=\frac{\pi}{2}$

22. (d) : $\sin^{-1}\left(\frac{2a}{1+a^2}\right)+\sin^{-1}\left(\frac{2b}{1+b^2}\right)=2\tan^{-1}x$

$\Rightarrow\ \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)+\sin^{-1}\left(\frac{2\tan\phi}{1+\tan^2\phi}\right)=2\tan^{-1}x$

[Putting $a=\tan\theta$ and $b=\tan\phi$]

$\Rightarrow\ \sin^{-1}\sin(2\theta)+\sin^{-1}\sin(2\phi)=2\tan^{-1}x$

$\Rightarrow\ 2(\theta+\phi)=2\tan^{-1}x$

Hence, $x=\tan(\theta+\phi)\Rightarrow x=\frac{\tan\theta+\tan\phi}{1-\tan\theta\tan\phi}=\frac{a+b}{1-ab}$

23. (c) : $4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{70}+\tan^{-1}\frac{1}{99}$

$=2\tan^{-1}\left[\frac{\frac{2}{5}}{1-\frac{1}{25}}\right]+\tan^{-1}\left[\frac{\frac{1}{99}-\frac{1}{70}}{1+\frac{1}{99}\cdot\frac{1}{70}}\right]$

$=2\tan^{-1}\left(\frac{5}{12}\right)+\tan^{-1}\left(\frac{-29}{6931}\right)$

$=\tan^{-1}\left[\frac{\frac{5}{6}}{1-\frac{25}{144}}\right]-\tan^{-1}\left(\frac{29}{6931}\right)$

$=\tan^{-1}\frac{120}{119}-\tan^{-1}\frac{1}{239}=\tan^{-1}(1)=\frac{\pi}{4}$

24. (b) : $\sin^{-1}x+\sin^{-1}y=\frac{2\pi}{3}$

$\Rightarrow\ \frac{\pi}{2}-\cos^{-1}x+\frac{\pi}{2}-\cos^{-1}y=\frac{2\pi}{3}$

$\Rightarrow\ \cos^{-1}x+\cos^{-1}y=\pi-\frac{2\pi}{3}=\frac{\pi}{3}$

25. (c) : $\tan^{-1}\frac{x}{y}-\tan^{-1}\left(\frac{x-y}{x+y}\right)$

$=\tan^{-1}\frac{x}{y}-\tan^{-1}\left(\frac{1-y/x}{1+y/x}\right)$

$=\tan^{-1}\frac{x}{y}-\left(\tan^{-1}1-\tan^{-1}\frac{y}{x}\right)=\tan^{-1}\frac{x}{y}+\tan^{-1}\frac{y}{x}-\frac{\pi}{4}$

$=\tan^{-1}\frac{x}{y}+\cot^{-1}\frac{x}{y}-\frac{\pi}{4}=\frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4}.$

26. (d) : $\sin^{-1}\frac{x}{5}+\operatorname{cosec}^{-1}\frac{5}{4}=\frac{\pi}{2}$

$\Rightarrow\ \sin^{-1}\frac{x}{5}=\frac{\pi}{2}-\sin^{-1}\frac{4}{5}\ \Rightarrow\ \sin^{-1}\frac{x}{5}=\cos^{-1}\frac{4}{5}$

$\Rightarrow\ \sin^{-1}\left(\frac{x}{5}\right)=\sin^{-1}\frac{3}{5}\ \Rightarrow\ x=3$

27. (b) : $\tan^{-1}\left(\frac{xy}{zr}\right)+\tan^{-1}\left(\frac{yz}{xr}\right)+\tan^{-1}\left(\frac{xz}{yr}\right)$

$=\tan^{-1}\left[\frac{\frac{xy}{zr}+\frac{yz}{xr}+\frac{xz}{yr}-\frac{xyz}{r^3}}{1-\left(\frac{x^2+y^2+z^2}{r^2}\right)}\right]=\tan^{-1}\infty=\frac{\pi}{2}$

28. (a) : We have $\frac{\pi^3}{32}\le(\sin^{-1}x)^3+(\cos^{-1}x)^3\le\frac{7\pi^3}{8}$

Here $a<\frac{1}{32}$. So, number of solution is zero.

29. (c) : $x+y=\tan^{-1}33$

$\Rightarrow\ y=\tan^{-1}33-\tan^{-1}3$ [$\because x=\tan^{-1}3$]

$\Rightarrow\ y=\tan^{-1}\left(\frac{33-3}{1+99}\right)=\tan^{-1}\left(\frac{30}{100}\right)\Rightarrow y=\tan^{-1}(0.3)$

30. (d) : $\tan^{-1}\left(\frac{1}{11}\right)+\tan^{-1}\left(\frac{2}{12}\right)$

$$=\tan^{-1}\left(\frac{12+22}{12\times 11-2}\right)=\tan^{-1}\left(\frac{17}{65}\right)$$

31. (c) : The given equation may be written as

$$\tan^{-1}x+\cot^{-1}x+\cot^{-1}x=\frac{2\pi}{3}$$

$$\Rightarrow \cot^{-1}x=\frac{2\pi}{3}-\frac{\pi}{2}=\frac{\pi}{6}\Rightarrow x=\sqrt{3}$$

32. (b) : We have, $4\sin^{-1}x+\cos^{-1}x=\pi$

$\Rightarrow 3\sin^{-1}x+\sin^{-1}x+\cos^{-1}x=\pi$

$$\Rightarrow 3\sin^{-1}x=\pi-\frac{\pi}{2}=\frac{\pi}{2}\Rightarrow \sin^{-1}x=\pi/6\Rightarrow x=\sin\frac{\pi}{6}=\frac{1}{2}$$

33. (a, b, d) : $2\cot^{-1}7+\cos^{-1}\frac{3}{5}=\cos^{-1}\left(\frac{44}{125}\right)$

34. (a,b,c): For $\tan^{-1}\frac{\sqrt{1-x^2}}{1+x}$, put $x=\cos\theta, 0<\theta<\frac{\pi}{2}$

35. (a) : $x^2+x+1=\lambda x+1, -1\le x^2+x+1\le 1$

$x=0$ or $x=\lambda-1, x^2+x\le 0, x^2+x+2\ge 0$

$\Rightarrow -1\le x\le 0$ and $\forall\ x\in R$

$\because$ $x=0$ is one solution

$-1\le x<0 \Rightarrow -1\le\lambda-1<0 \Rightarrow 0\le\lambda<1$

36. (a, c) : Domain of $f(x)=\ln\ \cos^{-1}x$ is $x\in[-1,1)$

$\therefore$ $[\alpha]=-1$ or 0

37. (a, c) : $f(x)=\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}\forall -1\le x\le 1$

$$\because\ -\frac{1}{2}\in[-1,1]\Rightarrow f\left(-\frac{1}{2}\right)=\frac{\pi}{2}$$

$$0<\frac{1}{1+k^2}\le 1\ \forall\ k\in R\Rightarrow f\left(\frac{1}{1+k^2}\right)=\frac{\pi}{2}$$

$k^2-2k+3=(k-1)^2+2\ge 2\ \forall\ k\in R$

38. (a) : The maximum value of $(\sin x+\cos x)$ is $\sqrt{2}$ while $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}\ \forall\ x\in[-1,1]$ so the equation $\sin x+\cos x=\sin^{-1}x+\cos^{-1}x$ has no solution.

39. (a) : $\sum_{i=1}^{2n}\sin^{-1}x_i=n\pi$ is possible if

$x_1=x_2=\ldots=x_{2n}=1$

$$\sum_{i=1}^{n}x_i=1+1+1+\ldots\ldots n\text{ terms}=n$$

$$\sum_{i=1}^{n}x_i^2=1^2+1^2+1^2+\ldots\ldots n\text{ terms}=n$$

$$\sum_{i=1}^{n}x_i^3=1^3+1^3+1^3+\ldots\ldots n\text{ terms}=n$$

40. (a) : Let $\sin\alpha=t\in(-1,0)$

So, $2\tan^{-1}\left(\frac{1}{t}\right)+\tan^{-1}\left(\frac{2t}{1-t^2}\right)$

$$=2\left[\tan^{-1}\left(\frac{1}{t}\right)+\tan^{-1}(t)\right]=2\left(-\frac{\pi}{2}\right)=-\pi$$

41. (a) : $\tan^{-1}x$ is an increasing function.

$\sec^{-1}x=\tan^{-1}\left(\sqrt{x^2-1}\right)$, for $x\ge 1$

Now, $\sec^{-1}x<\tan^{-1}x$ if $\tan^{-1}\left(\sqrt{x^2-1}\right)<\tan^{-1}x$

i.e., $\sqrt{x^2-1}<x$ *i.e.*, $x^2-1<x^2$, which is true.

42. (a) : $\sin^{-1}x=\tan^{-1}\frac{x}{\sqrt{1-x^2}}>\tan^{-1}x>\tan^{-1}y$

$$\left[\because\ x>y,\frac{x}{\sqrt{1-x^2}}>x\right]$$

$\therefore$ Statement -2 is true.

We know that $e<\pi\Rightarrow\frac{1}{\sqrt{e}}>\frac{1}{\sqrt{\pi}}$

So, by statement-2

$$\sin^{-1}\left(\frac{1}{\sqrt{e}}\right)>\tan^{-1}\left(\frac{1}{\sqrt{e}}\right)>\tan^{-1}\left(\frac{1}{\sqrt{\pi}}\right)$$

$\therefore$ Statement-1 is true.

43. (b) : $A=(\tan^{-1}x)^3+(\cot^{-1}x)^3$

$\Rightarrow A=(\tan^{-1}x+\cot^{-1}x)^3-3\tan^{-1}x\cot^{-1}x(\tan^{-1}x+\cot^{-1}x)$

$$\Rightarrow A=\left(\frac{\pi}{2}\right)^3-3\tan^{-1}x\cot^{-1}x\cdot\frac{\pi}{2}$$

$$\Rightarrow A=\frac{\pi^3}{32}+\frac{3\pi}{2}\left(\tan^{-1}x-\frac{\pi}{4}\right)^2$$

As $x>0$ $\therefore$ $\frac{\pi^3}{32}\le A<\frac{\pi^3}{8}$

44. (c) : $B=(\sin^{-1}t)^2+(\cos^{-1}t)^2$

$\Rightarrow B=(\sin^{-1}t+\cos^{-1}t)^2-2\sin^{-1}t\cos^{-1}t$

$$\Rightarrow B=\frac{\pi^2}{4}-2\sin^{-1}t\left(\frac{\pi}{2}-\sin^{-1}t\right)$$

$$\Rightarrow B=\frac{\pi^2}{8}+2\left(\sin^{-1}t-\frac{\pi}{4}\right)^2$$

$$\Rightarrow B_{\max}=\frac{\pi^2}{8}+2.\frac{\pi^2}{16}=\frac{\pi^2}{4}$$

45. (a) : We have, $\lambda=\frac{\pi^3}{32}$ and $\mu=\frac{\pi^2}{4}$

$$\therefore\ \frac{\lambda}{\mu}=\frac{\pi}{8}\text{ and }\frac{\lambda-\mu\pi}{\mu}=\frac{\pi}{8}-\pi=\frac{-7\pi}{8}$$

$$\therefore\ \cot^{-1}\cot\left(\frac{\lambda-\mu\pi}{\mu}\right)=\cot^{-1}\cot\left(-\frac{7\pi}{8}\right)=\frac{\pi}{8}$$

46. A → q ; B → p ; C → s ; D → r

(A) $\sin^{-1}(3x-4x^3)=\pi-3\sin^{-1}x$, if $\frac{1}{2}<x<1$

$$\therefore \lim_{x\to(1/2)^+} f(3x-4x^3)=\lim_{x\to(1/2)^+}(\pi-3\sin^{-1}x)$$
$$=\pi-3\lim_{x\to(1/2)^+}\sin^{-1}x$$
$\therefore$ $a = \pi \Rightarrow [a] = 3$

(B) $f(x)=\tan^{-1}\left(\dfrac{3x-x^3}{1-3x^2}\right)=3\tan^{-1}x,$

when $-\dfrac{1}{\sqrt{3}}<x<\dfrac{1}{\sqrt{3}}$

If $-\dfrac{1}{\sqrt{3}}<a<\dfrac{1}{\sqrt{3}},$

then $\lim\limits_{h\to0}\dfrac{f(a+3h)-f(a)}{3h}=\dfrac{3}{1+a^2}\Rightarrow f'(a)=\dfrac{3}{1+a^2}$

Now, $\lim\limits_{h\to0}\dfrac{f\left(\frac{1}{2}+6h\right)-f\left(\frac{1}{2}\right)}{6h}=f'(1/2)=\dfrac{12}{5}$

$\therefore$ Required value = 2

(C) $\cos^{-1}(4x^3-3x)=\cos^{-1}(\cos3\theta)=3\theta-2\pi$

$[\because 2\pi/3<\theta<\pi]$

$=-2\pi+3\cos^{-1}x$

$\therefore$ $[a+b+2]=[-2\pi+3+2]=-2$

(D) $f(x)=\cos^{-1}(4x^3-3x)=\cos^{-1}(\cos3\theta)$

$$=\begin{cases}3\theta, & 0<\theta<\dfrac{\pi}{3}\\ 2\pi-3\theta, & \dfrac{\pi}{3}<\theta<\dfrac{\pi}{2}\end{cases}=\begin{cases}3\cos^{-1}x, & \dfrac{1}{2}<x<1\\ 2\pi-3\cos^{-1}x, & 0<x<\dfrac{1}{2}\end{cases}$$

$$\therefore f'(x)=\begin{cases}\dfrac{-3}{\sqrt{1-x^2}}, & \dfrac{1}{2}<x<1\\ \dfrac{3}{\sqrt{1-x^2}}, & 0<x<\dfrac{1}{2}\end{cases}$$

$a=\lim\limits_{x\to(1/2)^+}f'(x)=-2\sqrt{3},\ b=\lim\limits_{x\to(1/2)^-}f'(x)=2\sqrt{3}$

$\therefore$ $[a+b-3]=-3$

47. (3) : Let $a=\cos^{-1}x$, $b=\sin^{-1}y$

$\therefore$ $a^2+b=1$ and $a+b^2=1$

$\Rightarrow$ $a=b$ or $a=1, b=0$ or $a=0, b=1$

If $a=b$, then $a^2+a-1=0\Rightarrow a=\dfrac{-1\pm\sqrt{5}}{2}$

$a\in(0,\pi), b\in\left(\dfrac{-\pi}{2},\dfrac{\pi}{2}\right)\Rightarrow a=b=\dfrac{\sqrt{5}-1}{2}$

$\Rightarrow (x,y)=\left(\cos\left(\dfrac{\sqrt{5}-1}{2}\right),\sin\left(\dfrac{\sqrt{5}-1}{2}\right)\right)$

$a=1, b=0 \Rightarrow (x,y)=(\cos1, 0)$

$a=0, b=1\Rightarrow(x,y)=(1,\sin 1)$

48. (1) : $S=\sum\limits_{r=1}^{50}(\tan^{-1}(1+r+r^2)-\tan^{-1}(1-r+r^2))$

$=\tan^{-1}(2551)-\tan^{-1}1=\tan^{-1}\left(\dfrac{2550}{2552}\right)=\tan^{-1}\left(\dfrac{1275}{1276}\right)$

$\therefore$ $\lambda=1275, \mu=1276\Rightarrow\mu-\lambda=1$

49. (1) : $P=\tan^{-1}\left[\dfrac{3\sin2\alpha}{5+3\cos2\alpha}\right]+\tan^{-1}\left[\dfrac{\tan\alpha}{4}\right]$

$=\tan^{-1}\left(\dfrac{6\tan\alpha}{8+2\tan^2\alpha}\right)+\tan^{-1}\left(\dfrac{\tan\alpha}{4}\right)$

$=\tan^{-1}\left(\dfrac{\dfrac{3\tan\alpha}{4+\tan^2\alpha}+\dfrac{\tan\alpha}{4}}{1-\dfrac{3\tan^2\alpha}{16+4\tan^2\alpha}}\right)\quad\left[\because\dfrac{3\tan^2\alpha}{16+4\tan^2\alpha}<1\right]$

$=\tan^{-1}(\tan\alpha)=\alpha$

50. (1) : $(\tan^{-1}x)^2+(\cot^{-1}x)^2=\dfrac{5\pi^2}{8}$

$\Rightarrow (\tan^{-1}x+\cot^{-1}x)^2-2\tan^{-1}x\left(\dfrac{\pi}{2}-\tan^{-1}x\right)=\dfrac{5\pi^2}{8}$

$\Rightarrow \dfrac{\pi^2}{4}-\pi\tan^{-1}x+2(\tan^{-1}x)^2=\dfrac{5\pi^2}{8}$

$\Rightarrow 2(\tan^{-1}x)^2-\pi\tan^{-1}x-\dfrac{3\pi^2}{8}=0$

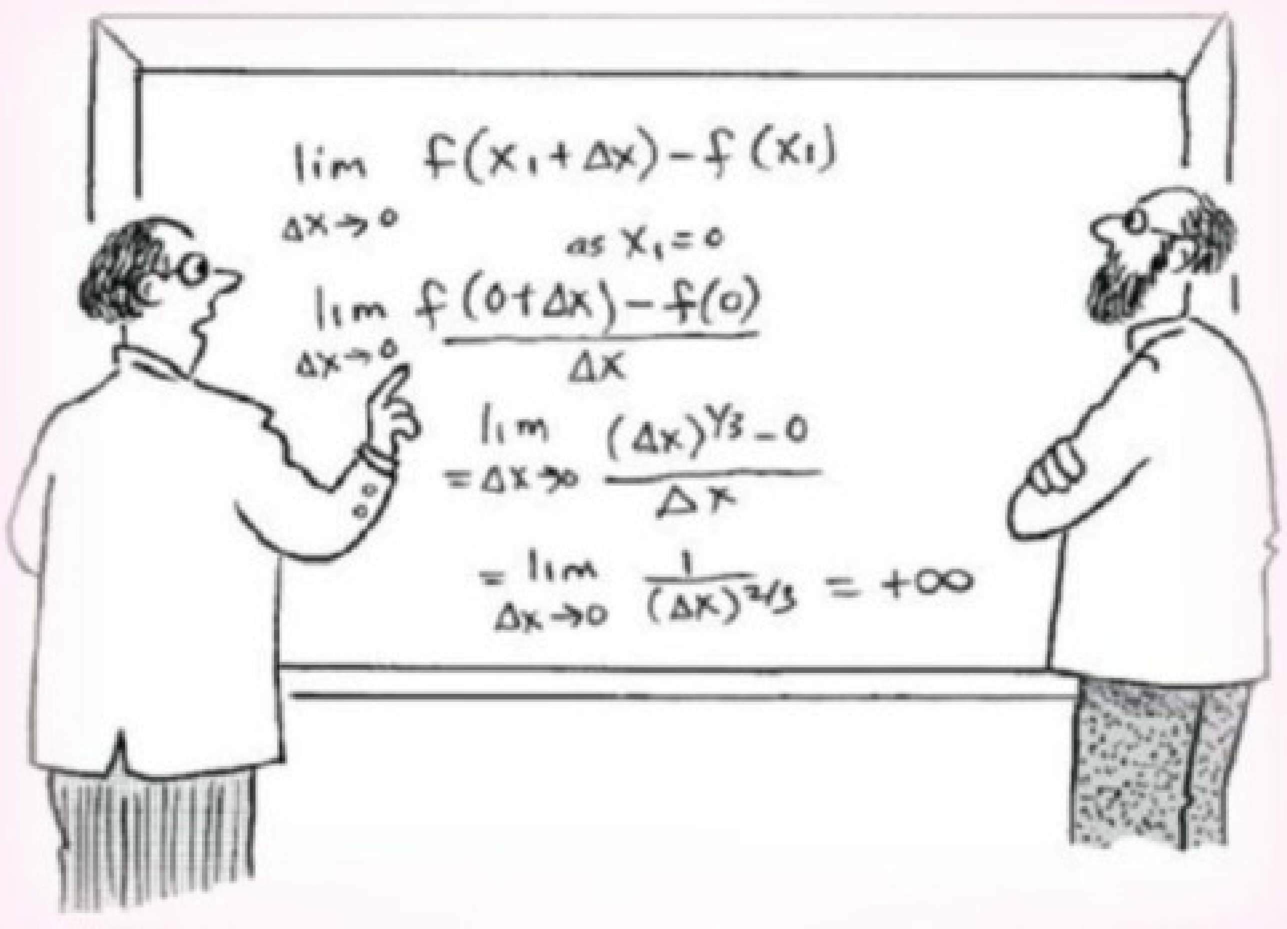
$\therefore$ $\tan^{-1}x=\dfrac{3\pi}{4},-\dfrac{\pi}{4}$

$\Rightarrow \tan^{-1}x=-\dfrac{\pi}{4}\quad\left[\because \tan^{-1}x\neq\dfrac{3\pi}{4}\right]$

$\therefore$ $x=-1$ is the solution.

◆

COMIC CAPSULE



"I proved that when you start to count your blessings, you find that they're infinite."

CBSE warm-up!

CLASS-XII

Chapterwise Practice questions for CBSE Exams as per the latest pattern and marking scheme issued by CBSE for the academic session 2019-20.

Series 7

Vector Algebra

Time Allowed : 3 hours
Maximum Marks : 80

GENERAL INSTRUCTIONS

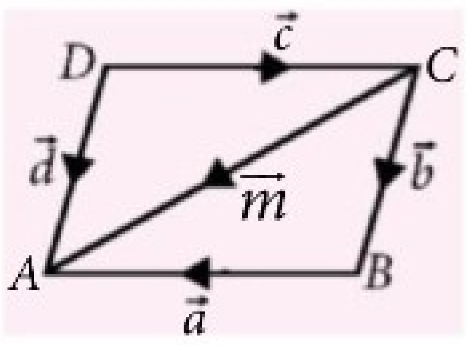
(i) All the questions are compulsory.
(ii) This question paper contains 36 questions.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
(iv) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

SECTION - A

Q.1-Q.10 are multiple choice type questions. Select the correct option.

1. The value of $(\hat{k}\times\hat{j})\cdot\hat{i}+\hat{j}\cdot\hat{k}$ is
(a) -1 (b) 1 (c) 0 (d) 2

2. In the given parallelogram, which of the following represents coinitial vectors?



(a) $\vec{c},\vec{d}$
(b) $\vec{m},\vec{b}$
(c) $\vec{b},\vec{d}$
(d) Both (a) and (b)

3. If O is origin and C is the mid point of $A(2, -1)$ and $B(-4, 3)$, then the value of $\overrightarrow{OC}$ is
(a) $\hat{i}+\hat{j}$ (b) $\hat{i}-\hat{j}$ (c) $-\hat{i}+\hat{j}$ (d) $-\hat{i}-\hat{j}$

4. If $|\vec{a}|=3$, $|\vec{b}|=4$, then the value of λ for which $\vec{a}+\lambda\vec{b}$ is perpendicular to $\vec{a}-\lambda\vec{b}$, is
(a) $\frac{9}{16}$ (b) $\frac{3}{4}$ (c) $\frac{3}{2}$ (d) $\frac{4}{3}$

5. If $\vec{a}$ and $\vec{b}$ are two non-zero vectors, then $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})$ is equal to
(a) $\vec{a}+\vec{b}$ (b) $\left|\vec{a}-\vec{b}\right|^2$
(c) $\left|\vec{a}+\vec{b}\right|^2$ (d) $|\vec{a}|^2-|\vec{b}|^2$

6. If $\vec{a}=x\hat{i}+2\hat{j}-z\hat{k}$ and $\vec{b}=3\hat{i}-y\hat{j}+\hat{k}$ are two equal vectors, then the value of $x + y + z$ is
(a) 0 (b) 1 (c) -1 (d) 2

7. A unit vector perpendicular to the plane of $\vec{a}=2\hat{i}-6\hat{j}-3\hat{k}$ and $\vec{b}=4\hat{i}+3\hat{j}-\hat{k}$ is
(a) $\frac{4\hat{i}+3\hat{j}-\hat{k}}{\sqrt{26}}$ (b) $\frac{2\hat{i}-6\hat{j}-3\hat{k}}{7}$
(c) $\frac{3\hat{i}-2\hat{j}+6\hat{k}}{7}$ (d) $\frac{2\hat{i}-3\hat{j}-6\hat{k}}{7}$

8. The two vectors $\vec{a}=2\hat{i}+\hat{j}+3\hat{k}$ and $\vec{b}=4\hat{i}-\lambda\hat{j}+6\hat{k}$ are parallel, if λ is equal to
(a) 2 (b) -3 (c) 3 (d) -2

9. If $\vec{a}=7\hat{i}+\hat{j}-4\hat{k}$ and $\vec{b}=2\hat{i}+6\hat{j}+3\hat{k}$, then the projection of $\vec{a}$ on $\vec{b}$ is

(a) $\frac{4}{7}$ (b) $\frac{8}{7}$ (c) $\frac{6}{7}$ (d) $\frac{2}{7}$

10. If the vectors $2\hat{i}-3\hat{j}$, $\hat{i}+\hat{j}-\hat{k}$ and $3\hat{i}-\hat{k}$ form three concurrent edges of a parallelopiped, then the volume of the parallelopiped (in cubic units) is

(a) 8 (b) 10 (c) 4 (d) 14

(Q.11 - Q.15) Fill in the blanks.

11. If $\vec{r}\cdot\vec{a}=0$, $\vec{r}\cdot\vec{b}=0$ and $\vec{r}\cdot\vec{c}=0$ for some non-zero vector $\vec{r}$, then the value of $\vec{a}\cdot(\vec{b}\times\vec{c})$ is ______.

OR

The area of the parallelogram whose adjacent sides are determined by the vector $\vec{a}=\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}-7\hat{j}+\hat{k}$ is ______ square units.

12. The vector having initial and terminal point as (2, 5, 0) and (–3, 7, 4) respectively is ______.

13. If $|\vec{a}|=10$, $|\vec{b}|=2$ and $\vec{a}\cdot\vec{b}=12$, then the sine of angle between $\vec{a}$ and $\vec{b}$ is ______.

14. The sum of the vectors $\vec{a}=\hat{i}-2\hat{j}+\hat{k}$, $\vec{b}=-2\hat{i}+4\hat{j}+5\hat{k}$ and $\vec{c}=\hat{i}-6\hat{j}-7\hat{k}$ is ______.

OR

The value of 'p' for which the vectors $3\hat{i}+2\hat{j}+9\hat{k}$ and $\hat{i}-2p\hat{j}+3\hat{k}$ are parallel is ______.

15. The angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2 respectively having $\vec{a}\cdot\vec{b}=\sqrt{6}$ is ______.

(Q.16-Q.20) Answer the following questions.

16. If the position vector $\vec{a}$ of a point (12, n) is such that $|\vec{a}|=13$, find the value of n.

17. Find the value of λ so that the vectors $\vec{a}=2\hat{i}+\lambda\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+3\hat{k}$ are perpendicular to each other.

18. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $(2\vec{a}+\vec{b})$ and $(\vec{a}-3\vec{b})$ respectively, externally in the ratio 1 : 2.

OR

If $|\vec{a}|=2$, $|\vec{b}|=7$ and $(\vec{a}\times\vec{b})=3\hat{i}+2\hat{j}+6\hat{k}$, find the angle between $\vec{a}$ and $\vec{b}$.

19. Find the angle between x-axis and the vector $\hat{i}+\hat{j}+\hat{k}$.

20. Vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}|=3, |\vec{b}|=\frac{2}{3}$ and $(\vec{a}\times\vec{b})$ is a unit vector. Write the angle between $\vec{a}$ and $\vec{b}$.

SECTION - B

21. If A, B and C have position vectors (2, 0, 0), (0, 1, 0) and (0, 0, 2) respectively, show that ΔABC is isosceles.

OR

If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are mutually perpendicular unit vectors, then find the value of $|2\hat{a}+\hat{b}+\hat{c}|$.

22. Vectors drawn from the origin O to the points A, B and C are respectively $\vec{a}$, $\vec{b}$ and $4\vec{a}-3\vec{b}$. Find $\overrightarrow{AC}$ and $\overrightarrow{BC}$.

23. If $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$, show that the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular to each other.

24. Find a unit vector parallel to the sum of the vectors $\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}-3\hat{j}+5\hat{k}$.

OR

If $\vec{a}$ and $\vec{b}$ are unit vectors, then what is the angle between $\vec{a}$ and $\vec{b}$ so that $\sqrt{2}\,\vec{a}-\vec{b}$ is a unit vector?

25. Show that the area of the parallelogram having diagonals $3\hat{i}+\hat{j}-2\hat{k}$ and $\hat{i}-3\hat{j}+4\hat{k}$ is $5\sqrt{3}$ square units.

26. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, show that $(\vec{a}-\vec{d})$ is parallel to $(\vec{b}-\vec{c})$, it is being given that $\vec{a}\neq\vec{d}$ and $\vec{b}\neq\vec{c}$.

SECTION - C

27. The scalar product of the vector $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $\vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$ and $\vec{c}=\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ and hence find the unit vector along $\vec{b}+\vec{c}$.

28. Vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=3$, $|\vec{b}|=5$ and $|\vec{c}|=7$.
Find the angle between $\vec{a}$ and $\vec{b}$.

OR

If $\vec{a}=3\hat{i}-\hat{j}+2\hat{k}, \vec{b}=2\hat{i}+\hat{j}-\hat{k}, \vec{c}=\hat{i}-2\hat{j}+2\hat{k}$, find $(\vec{a}\times\vec{b})\times\vec{c}$ and $\vec{a}\times(\vec{b}\times\vec{c})$ and hence show that $(\vec{a}\times\vec{b})\times\vec{c}\neq\vec{a}\times(\vec{b}\times\vec{c})$.

29. Let $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}, \vec{b}=-\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}$. Find a unit vector in the direction of resultant of these vectors. Also, find a vector $\vec{r}$ which is normal to both $\vec{a}$ and $\vec{b}$. What is the inclination of $\vec{r}$ to $\vec{c}$?

30. Let $\vec{a}=4\hat{i}+5\hat{j}-\hat{k}, \vec{b}=\hat{i}-4\hat{j}+5\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}-\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{c}$ and $\vec{b}$ and $\vec{d}\cdot\vec{a}=21$.

31. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=4\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{c}=\hat{i}-2\hat{j}+\hat{k}$, then find a vector of magnitude 6 units which is parallel to the vector $2\vec{a}-\vec{b}+3\vec{c}$.

OR

Write the projection of $\vec{b}+\vec{c}$ on $\vec{a}$, where $\vec{a}=2\hat{i}-2\hat{j}+\hat{k}, \vec{b}=\hat{i}+2\hat{j}-2\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$.

32. If $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}, \vec{a}\times\vec{b}=\vec{a}\times\vec{c}$ and $\vec{a}\neq\vec{0}$, then prove that $\vec{b}=\vec{c}$.

SECTION - D

33. If $\vec{a},\vec{b},\vec{c}$ are non-coplanar vectors, then prove that $\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}$ are non-coplanar.

OR

If $\vec{a}=\hat{i}-\hat{j}-3\hat{k}, \vec{b}=4\hat{i}-3\hat{j}+\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+2\hat{k}$, verify that $\vec{a}\times(\vec{b}+\vec{c})=\vec{a}\times\vec{b}+\vec{a}\times\vec{c}$.

34. Using vectors find the area of ΔABC whose vertices are $A(1, 2, 3)$, $B(2, -1, 4)$ and $C(4, 5, -1)$.

35. The two adjacent sides of a parallelogram are $2\hat{i}-4\hat{j}-5\hat{k}$ and $2\hat{i}+2\hat{j}+3\hat{k}$. Find the two unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram.

OR

The lines joining the vertices of a tetrahedron to the centroids of opposite faces are concurrent.

36. Find the value of λ, if four points with position vectors $3\hat{i}+6\hat{j}+9\hat{k}$, $\hat{i}+2\hat{j}+3\hat{k}$, $2\hat{i}+3\hat{j}+\hat{k}$ and $4\hat{i}+6\hat{j}+\lambda\hat{k}$ are coplanar.

SOLUTIONS

1. **(a) :** $(\hat{k}\times\hat{j})\cdot\hat{i}+\hat{j}\cdot\hat{k}=-\hat{i}\cdot\hat{i}+\hat{j}\cdot\hat{k}=-1+0=-1$

2. **(d) :** $(\vec{c},\vec{d})$ and $(\vec{m},\vec{b})$ represents coinitial vectors.

3. **(c)**

4. **(b) :** Given that, $|\vec{a}|=3$, $|\vec{b}|=4$ and $\vec{a}+\lambda\vec{b}$ is perpendicular to $\vec{a}-\lambda\vec{b}$.

$\therefore (\vec{a}+\lambda\vec{b})\cdot(\vec{a}-\lambda\vec{b})=0\Rightarrow\vec{a}\cdot\vec{a}-\lambda\vec{a}\cdot\vec{b}+\lambda\vec{b}\cdot\vec{a}-\lambda^2\vec{b}\cdot\vec{b}=0$

$$\Rightarrow |\vec{a}|^2-\lambda^2|\vec{b}|^2=0 \Rightarrow \lambda^2=\frac{|\vec{a}|^2}{|\vec{b}|^2}\Rightarrow\lambda=\frac{|\vec{a}|}{|\vec{b}|}=\frac{3}{4}$$

5. **(d) :** $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})=\vec{a}\cdot\vec{a}-\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{a}-\vec{b}\cdot\vec{b}$

$$=|\vec{a}|^2-|\vec{b}|^2$$

6. **(a) :** Given, $\vec{a}=\vec{b} \Rightarrow x\hat{i}+2\hat{j}-z\hat{k}=3\hat{i}-y\hat{j}+\hat{k}$

$\Rightarrow x=3, y=-2, z=-1 \therefore x+y+z=3+(-2)+(-1)=0$

7. **(c) :** $\vec{a}\times\vec{b}=15\hat{i}-10\hat{j}+30\hat{k}$

and $|\vec{a}\times\vec{b}|=\sqrt{15^2+(-10)^2+(30)^2}=35$

$\therefore$ Required vector $=\dfrac{3\hat{i}-2\hat{j}+6\hat{k}}{7}$

8. **(d) :** For parallel vectors, $\vec{a}\times\vec{b}=\vec{0}$

$$\Rightarrow \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & 3\\ 4 & -\lambda & 6\end{vmatrix}=0 \Rightarrow \hat{i}(6+3\lambda)-\hat{j}(0)+\hat{k}(-2\lambda-4)=0$$

$\therefore$ $6+3\lambda=0$ and $-2\lambda-4=0 \Rightarrow \lambda=-2$.

9. **(b) :** Projection of $\vec{a}$ on $\vec{b}=\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$$=\frac{(7\hat{i}+\hat{j}-4\hat{k})\cdot(2\hat{i}+6\hat{j}+3\hat{k})}{\sqrt{(2)^2+(6)^2+(3)^2}}=\frac{14+6-12}{7}=\frac{8}{7}$$

10. **(c)**

11. Since, $\vec{r}$ is a non-zero vector and $\vec{r}\cdot\vec{a}=\vec{r}\cdot\vec{b}=\vec{r}\cdot\vec{c}=0$.

So, $\vec{a},\vec{b}$ and $\vec{c}$ are coplanar.

$\therefore$ $\vec{a}\cdot(\vec{b}\times\vec{c})=0$

OR

$$\text{Here, } \vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 3\\ 2 & -7 & 1\end{vmatrix}$$

$$=(-1+21)\hat{i}-(1-6)\hat{j}+(-7+2)\hat{k}=20\hat{i}+5\hat{j}-5\hat{k}$$

$\therefore$ Area of parallelogram $=|\vec{a}\times\vec{b}|$

$$=\sqrt{(20)^2+(5)^2+(-5)^2}=15\sqrt{2} \text{ sq. units.}$$

12. Let $A(2, 5, 0)$ and $B(-3, 7, 4)$

$\therefore$ Required vector $=(-3-2)\hat{i}+(7-5)\hat{j}+(4-0)\hat{k}$

$$=-5\hat{i}+2\hat{j}+4\hat{k}$$

13. Given, $|\vec{a}|=10$, $|\vec{b}|=2$, $\vec{a}\cdot\vec{b}=12$

We know, $\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow$ $12=10\times2\cos\theta \Rightarrow \cos\theta=\dfrac{3}{5} \quad\therefore\quad \sin\theta=\dfrac{4}{5}$

14. The given vectors are

$\vec{a}=\hat{i}-2\hat{j}+\hat{k}, \vec{b}=-2\hat{i}+4\hat{j}+5\hat{k}, \vec{c}=\hat{i}-6\hat{j}-7\hat{k}$

$\therefore$ Their sum $=\vec{a}+\vec{b}+\vec{c}$

$=(\hat{i}-2\hat{j}+\hat{k})+(-2\hat{i}+4\hat{j}+5\hat{k})+(\hat{i}-6\hat{j}-7\hat{k})=-4\hat{j}-\hat{k}$.

OR

$\vec{a}=3\hat{i}+2\hat{j}+9\hat{k}$ and $\vec{b}=\hat{i}-2p\hat{j}+3\hat{k}$

For $\vec{a}$ and $\vec{b}$ to be parallel, $\vec{b}=\lambda\vec{a}$.

$\Rightarrow \hat{i}-2p\hat{j}+3\hat{k}=\lambda(3\hat{i}+2\hat{j}+9\hat{k})=3\lambda\hat{i}+2\lambda\hat{j}+9\lambda\hat{k}$

$\Rightarrow 1=3\lambda; -2p=2\lambda, 3=9\lambda \Rightarrow \lambda=\dfrac{1}{3}$ and $p=-\lambda=-\dfrac{1}{3}$

15. Let θ be the angle between $\vec{a}$ and $\vec{b}$, then

$$\cos\theta=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}=\frac{\sqrt{6}}{\sqrt{3}\cdot 2}\times\frac{\sqrt{3}}{\sqrt{3}}$$

$$=\frac{\sqrt{18}}{3\times 2}=\frac{3\sqrt{2}}{3\times 2}=\frac{\sqrt{2}}{2}=\frac{1}{\sqrt{2}}\quad\therefore\quad \theta=\cos^{-1}\frac{1}{\sqrt{2}}\Rightarrow\theta=\frac{\pi}{4}.$$

16. The position vector of the point (12, n) is $12\hat{i}+n\hat{j}$.

$\therefore\quad \vec{a}=12\hat{i}+n\hat{j}\Rightarrow |\vec{a}|=\sqrt{12^2+n^2}$

But, $|\vec{a}|=13\ \therefore\ 13=\sqrt{12^2+n^2}$

$\Rightarrow\ 169=144+n^2\Rightarrow n^2=25\Rightarrow n=\pm 5$

17. If the vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other, then $\vec{a}\cdot\vec{b}=0$

$\Rightarrow\ (2\hat{i}+\lambda\hat{j}+\hat{k})\cdot(\hat{i}-2\hat{j}+3\hat{k})=0$

$\Rightarrow\ (2)(1)+\lambda(-2)+(1)(3)=0\Rightarrow -2\lambda+5=0\Rightarrow\lambda=\frac{5}{2}.$

18. The position vector of R is given by

$$\vec{r}=\frac{1(\vec{a}-3\vec{b})-2(2\vec{a}+\vec{b})}{1-2}=\frac{-3\vec{a}-5\vec{b}}{-1}=3\vec{a}+5\vec{b}$$

OR

Given, $|\vec{a}|=2,\ |\vec{b}|=7$ and $\vec{a}\times\vec{b}=3\hat{i}+2\hat{j}+6\hat{k}$

$\therefore\quad |\vec{a}\times\vec{b}|=\sqrt{3^2+2^2+6^2}=\sqrt{49}=7$

Let θ be the angle between $\vec{a}$ and $\vec{b}$

Now, $\sin\theta=\dfrac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}=\dfrac{7}{2\times 7}=\dfrac{1}{2}\quad\therefore\quad\theta=\dfrac{\pi}{6}$

19. Here, $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and vector along x-axis is $\hat{i}$.

$\therefore$ Angle between $\vec{a}$ and $\hat{i}$ is given by

$$\cos\theta=\frac{\vec{a}\cdot\hat{i}}{\sqrt{1^2+1^2+1^2}\cdot\sqrt{1^2}}=\frac{(\hat{i}+\hat{j}+\hat{k})\cdot\hat{i}}{\sqrt{3}\cdot 1}=\frac{1}{\sqrt{3}}$$

$$\Rightarrow\theta=\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

20. Given, $|\vec{a}|=3, |\vec{b}|=\frac{2}{3}, |\vec{a}\times\vec{b}|=1$

$\Rightarrow |\vec{a}||\vec{b}|\sin\theta=1\ \Rightarrow 3.\frac{2}{3}\sin\theta=1\ \Rightarrow\theta=\frac{\pi}{6}$

21. We have, $\overrightarrow{AB}$ = P.V. of B – P.V. of A

$\Rightarrow\ \overrightarrow{AB}=(0\hat{i}+\hat{j}+0\hat{k})-(2\hat{i}+0\hat{j}+0\hat{k})=-2\hat{i}+\hat{j}+0\hat{k}$

$\therefore\ |\overrightarrow{AB}|=\sqrt{(-2)^2+1^2+0^2}=\sqrt{5}$

$\overrightarrow{BC}$ = P.V. of C – P.V. of B

$\Rightarrow\ \overrightarrow{BC}=(0\hat{i}+0\hat{j}+2\hat{k})-(0\hat{i}+\hat{j}+0\hat{k})=0\hat{i}-\hat{j}+2\hat{k}$

$\therefore\ |\overrightarrow{BC}|=\sqrt{0^2+(-1)^2+2^2}=\sqrt{5}$

Clearly, $AB=BC$. Hence, ΔABC is isosceles.

OR

Here $\hat{a},\hat{b}$ and $\hat{c}$ are mutually perpendicular unit vectors.

$\therefore\ |\hat{a}|=|\hat{b}|=|\hat{c}|=1$ and $\hat{a}\cdot\hat{b}=\hat{b}\cdot\hat{c}=\hat{c}\cdot\hat{a}=0$ ...(i)

Now, $|2\hat{a}+\hat{b}+\hat{c}|^2=(2\hat{a}+\hat{b}+\hat{c})\cdot(2\hat{a}+\hat{b}+\hat{c})$

$=4\hat{a}\cdot\hat{a}+2\hat{a}\cdot\hat{b}+2\hat{a}\cdot\hat{c}+2\hat{b}\cdot\hat{a}+\hat{b}\cdot\hat{b}+\hat{b}\cdot\hat{c}+2\hat{c}\cdot\hat{a}+\hat{c}\cdot\hat{b}+\hat{c}\cdot\hat{c}$

$=4|\hat{a}|^2+|\hat{b}|^2+|\hat{c}|^2+4\hat{a}\cdot\hat{b}+2\hat{b}\cdot\hat{c}+4\hat{a}\cdot\hat{c}$

$(\because\ \hat{b}\cdot\hat{a}=\hat{a}\cdot\hat{b},\hat{c}\cdot\hat{a}=\hat{a}\cdot\hat{c},\hat{c}\cdot\hat{b}=\hat{b}\cdot\hat{c})$

$=4\cdot 1^2+1^2+1^2=6$ [Using (i)]

$\therefore\ |2\hat{a}+\hat{b}+\hat{c}|=\sqrt{6}.$

22. In ΔOAC, we have

$\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}\ =4\vec{a}-3\vec{b}-\vec{a}\ =3(\vec{a}-\vec{b})$

In ΔOBC, we have

$\Rightarrow\ \overrightarrow{BC}=\overrightarrow{OC}-\overrightarrow{OB}\ =4\vec{a}-3\vec{b}-\vec{b}=4(\vec{a}-\vec{b}).$

23. We have, $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$,

$\therefore\quad \vec{a}+\vec{b}=4\hat{i}+\hat{j}-\hat{k}$ and $\vec{a}-\vec{b}=-2\hat{i}+3\hat{j}-5\hat{k}$

$\therefore\quad (\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})=4(-2)+(1\times 3)+(-1)(-5)=0.$

Therefore, the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular to each other.

24. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}-3\hat{j}+5\hat{k}$

$\therefore \vec{a}+\vec{b}=(\hat{i}+\hat{j}+\hat{k})+(2\hat{i}-3\hat{j}+5\hat{k})=3\hat{i}-2\hat{j}+6\hat{k}$

Any vector parallel to $\vec{a}+\vec{b}$

$=\lambda(\vec{a}+\vec{b})=\lambda(3\hat{i}-2\hat{j}+6\hat{k})$

$\therefore$ The unit vector in this direction

$$=\frac{\lambda(3\hat{i}-2\hat{j}+6\hat{k})}{\sqrt{(3\lambda)^2+(-2\lambda)^2+(6\lambda)^2}}=\pm\frac{1}{7}(3\hat{i}-2\hat{j}+6\hat{k})$$

OR

Let θ be the angle between the unit vectors $\vec{a}$ and $\vec{b}$.

$\therefore\ \cos\theta=\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}=\vec{a}\cdot\vec{b}\qquad(\because\ |\vec{a}|=1=|\vec{b}|)$ ...(i)

Now, $1=\left|\sqrt{2}\,\vec{a}-\vec{b}\right|$

$\Rightarrow\ 1=\left|\sqrt{2}\,\vec{a}-\vec{b}\right|^2=(\sqrt{2}\,\vec{a}-\vec{b})\cdot(\sqrt{2}\,\vec{a}-\vec{b})$

$=2|\vec{a}|^2-\sqrt{2}\,\vec{a}\cdot\vec{b}-\vec{b}\cdot\sqrt{2}\,\vec{a}+|\vec{b}|^2=2-2\sqrt{2}\,\vec{a}\cdot\vec{b}+1$

$(\because \vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a})$

$\Rightarrow\ 1=3-2\sqrt{2}\,\vec{a}\cdot\vec{b}$

$\Rightarrow\ \vec{a}\cdot\vec{b}=\dfrac{1}{\sqrt{2}}\Rightarrow\ \cos\theta=\dfrac{1}{\sqrt{2}}$ [By using (i)]

$\therefore\quad\theta=\pi/4$

25. We know, area of parallelogram with diagonals as $\vec{d}_1$ and $\vec{d}_2$ is given by $\frac{1}{2}|\overrightarrow{d_1}\times\overrightarrow{d_2}|$

Here, $\overrightarrow{d_1}=3\hat{i}+\hat{j}-2\hat{k}$, $\overrightarrow{d_2}=\hat{i}-3\hat{j}+4\hat{k}$

Now, $\overrightarrow{d_1}\times\overrightarrow{d_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 1 & -2\\ 1 & -3 & 4\end{vmatrix}$

$= \hat{i}(4-6)-\hat{j}(12+2)+\hat{k}(-9-1) = -2\hat{i}-14\hat{j}-10\hat{k}$

$|\overrightarrow{d_1}\times\overrightarrow{d_2}|=\sqrt{(-2)^2+(-14)^2+(-10)^2}=\sqrt{300}=10\sqrt{3}$

$\therefore$ Area of parallelogram $=\frac{1}{2}(10\sqrt{3})=5\sqrt{3}$ square units.

26. Given, $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$ ...(i)

Now, $(\vec{a}-\vec{d})\times(\vec{b}-\vec{c})=(\vec{a}\times\vec{b})-(\vec{d}\times\vec{b})-(\vec{a}\times\vec{c})+(\vec{d}\times\vec{c})$ [By distributive law]

$=(\vec{a}\times\vec{b})+(\vec{b}\times\vec{d})-(\vec{a}\times\vec{c})-(\vec{c}\times\vec{d})=0$ [Using (i)]

$\Rightarrow (\vec{a}-\vec{d})\times(\vec{b}-\vec{c})=\vec{0} \Rightarrow (\vec{a}-\vec{d})\,||\,(\vec{b}-\vec{c})$

[Here $|\vec{a}-\vec{d}|\neq 0$ and $|\vec{b}-\vec{c}|\neq 0$ as $\vec{a}\neq\vec{d}$ and $\vec{b}\neq\vec{c}$].

27. *Refer to answer 90, Page no. 259 of MTG CBSE Champion Mathematics, Class-12*

28. $\vec{a}+\vec{b}+\vec{c}=0$ ...(i)

Also, $|\vec{a}|=3, |\vec{b}|=5, |\vec{c}|=7$ ...(ii)

From (i), we have $\vec{a}+\vec{b}=-\vec{c} \Rightarrow |\vec{a}+\vec{b}|^2=|-\vec{c}|^2$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+2(\vec{a}\cdot\vec{b})=|\vec{c}|^2$

$\Rightarrow 9+25+2|\vec{a}||\vec{b}|\cos\theta=49$ [Using (ii)]

$\Rightarrow 2\times 3\times 5\times\cos\theta = 49-34=15$

$\Rightarrow \cos\theta=\frac{15}{30}=\frac{1}{2} \Rightarrow \theta=\frac{\pi}{3}=60^\circ$

OR

$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -1 & 2\\ 2 & 1 & -1\end{vmatrix}=-\hat{i}+7\hat{j}+5\hat{k}$

$\therefore (\vec{a}\times\vec{b})\times\vec{c}=(-\hat{i}+7\hat{j}+5\hat{k})\times(\hat{i}-2\hat{j}+2\hat{k})$

$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 7 & 5\\ 1 & -2 & 2\end{vmatrix}=24\hat{i}+7\hat{j}-5\hat{k}$ ...(i)

Again, $\vec{b}\times\vec{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & -1\\ 1 & -2 & 2\end{vmatrix}=-5\hat{j}-5\hat{k}$

$\therefore \vec{a}\times(\vec{b}\times\vec{c})=(3\hat{i}-\hat{j}+2\hat{k})\times(-5\hat{j}-5\hat{k})$

$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -1 & 2\\ 0 & -5 & -5\end{vmatrix}=15(\hat{i}+\hat{j}-\hat{k})$ ...(ii)

From (i) and (ii), we conclude that

$(\vec{a}\times\vec{b})\times\vec{c}\neq\vec{a}\times(\vec{b}\times\vec{c})$

29. The resultant of given vectors is given by

$(\vec{a}+\vec{b}+\vec{c})=3\hat{i}+5\hat{j}+4\hat{k}$

$\therefore |\vec{a}+\vec{b}+\vec{c}|=\sqrt{3^2+5^2+4^2}=5\sqrt{2}$

$\therefore$ Unit vector in the direction of the resultant

$=\frac{1}{5\sqrt{2}}(3\hat{i}+5\hat{j}+4\hat{k})$

Let $\vec{r}=\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 3\\ -1 & 2 & 1\end{vmatrix}=-4\hat{i}-4\hat{j}+4\hat{k}$

$\therefore |\vec{r}|=\sqrt{(-4)^2+(-4)^2+4^2}=4\sqrt{3}$

Let θ be the angle between $\vec{r}$ and $\vec{c}$, then

$\theta=\cos^{-1}\left\{\frac{\vec{r}\cdot\vec{c}}{|\vec{r}||\vec{c}|}\right\}$

$=\cos^{-1}\left\{\frac{(-4\hat{i}-4\hat{j}+4\hat{k})\cdot(3\hat{i}+\hat{j})}{(4\sqrt{3})\cdot(\sqrt{10})}\right\}$ or $\theta=\cos^{-1}\left(\frac{-4}{\sqrt{30}}\right)$

$\therefore$ Angle between $\vec{r}$ and $\vec{c}=\pi-\cos^{-1}\left(\frac{4}{\sqrt{30}}\right)$

30. *Refer to answer 82, Page no. 257 of MTG CBSE Champion Mathematics, Class-12*

31. We have, $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=4\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{c}=\hat{i}-2\hat{j}+\hat{k}$

$\therefore 2\vec{a}-\vec{b}+3\vec{c}=2(\hat{i}+\hat{j}+\hat{k})-(4\hat{i}-2\hat{j}+3\hat{k})+3(\hat{i}-2\hat{j}+\hat{k})$

$=\hat{i}-2\hat{j}+2\hat{k}$

$\Rightarrow |2\vec{a}-\vec{b}+3\vec{c}|=\sqrt{1^2+(-2)^2+2^2}=3$

Now, a vector of magnitude 6 units in direction of

$2\vec{a}-\vec{b}+3\vec{c}=\frac{6}{3}\left(\hat{i}-2\hat{j}+2\hat{k}\right)=2\hat{i}-4\hat{j}+4\hat{k}$

OR

Here, $\vec{a}=2\hat{i}-2\hat{j}+\hat{k}, \vec{b}=\hat{i}+2\hat{j}-2\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$

$\Rightarrow \vec{b}+\vec{c}=3\hat{i}+\hat{j}+2\hat{k}$

Now, projection of $\vec{b}+\vec{c}$ on $\vec{a}$

$=\frac{(\vec{b}+\vec{c})\cdot\vec{a}}{|\vec{a}|}=\frac{(3\hat{i}+\hat{j}+2\hat{k})\cdot(2\hat{i}-2\hat{j}+\hat{k})}{|2\hat{i}-2\hat{j}+\hat{k}|}$

$=\frac{3\times 2+1\times(-2)+2\times 1}{\sqrt{2^2+(-2)^2+1^2}}=\frac{6}{3}=2$

32. Given, $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c} \Rightarrow \vec{a}\cdot\vec{b}-\vec{a}\cdot\vec{c}=\vec{0}$

$\Rightarrow \vec{a}\cdot(\vec{b}-\vec{c})=0 \Rightarrow \vec{b}-\vec{c}=\vec{0}$ or $\vec{a}\perp(\vec{b}-\vec{c})$

$\Rightarrow \vec{b}=\vec{c}$ or $\vec{a}\perp(\vec{b}-\vec{c})$ ...(i)

Again, given, $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}$ and $\vec{a}\neq\vec{0}$

$\Rightarrow (\vec{a}\times\vec{b})-(\vec{a}\times\vec{c})=\vec{0} \Rightarrow \vec{a}\times(\vec{b}-\vec{c})=\vec{0}$

$\Rightarrow (\vec{b}-\vec{c})=\vec{0}$ or $\vec{a}\,||\,(\vec{b}-\vec{c})$

$\Rightarrow \quad \vec{b}=\vec{c} \quad \text{or} \quad \vec{a}\,||\,(\vec{b}-\vec{c})$ ...(ii)

From (i) and (ii), we get $\vec{b}=\vec{c}$

[$\because$ $\vec{a}\perp(\vec{b}-\vec{c})$ and $\vec{a}\,||\,(\vec{b}-\vec{c})$ both cannot be simultaneously true]

33. Consider $[\vec{a}+\vec{b} \;\; \vec{b}+\vec{c} \;\; \vec{c}+\vec{a}]$

$$=(\vec{a}+\vec{b})\cdot[(\vec{b}+\vec{c})\times(\vec{c}+\vec{a})]$$

$$=(\vec{a}+\vec{b})\cdot[(\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{c}+\vec{c}\times\vec{a})]$$

$$=(\vec{a}+\vec{b})\cdot[\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{a}] \quad [\because\ \vec{c}\times\vec{c}=\vec{0}]$$

$$=\vec{a}\cdot(\vec{b}\times\vec{c})+\vec{a}\cdot(\vec{b}\times\vec{a})+\vec{a}\cdot(\vec{c}\times\vec{a})+\vec{b}\cdot(\vec{b}\times\vec{c})+\vec{b}\cdot(\vec{b}\times\vec{a})+\vec{b}\cdot(\vec{c}\times\vec{a})$$

$$=[\vec{a}\;\vec{b}\;\vec{c}]+[\vec{a}\;\vec{b}\;\vec{a}]+[\vec{a}\;\vec{c}\;\vec{a}]+[\vec{b}\;\vec{b}\;\vec{c}]+[\vec{b}\;\vec{b}\;\vec{a}]+[\vec{b}\;\vec{c}\;\vec{a}]$$

$$=[\vec{a}\;\vec{b}\;\vec{c}]+[\vec{b}\;\vec{c}\;\vec{a}]=2\,[\vec{a}\;\vec{b}\;\vec{c}] \qquad \text{...(i)}$$

Given, $\vec{a},\vec{b},\vec{c}$ are non-coplanar vectors

$\Rightarrow \quad [\vec{a}\;\vec{b}\;\vec{c}]\neq 0$ ...(ii)

$\Rightarrow \quad [\vec{a}+\vec{b} \;\; \vec{b}+\vec{c} \;\; \vec{c}+\vec{a}]\neq 0$ [From (i)]

Thus, $\vec{a}+\vec{b},\vec{b}+\vec{c},\vec{c}+\vec{a}$ are non-coplanar vectors.

OR

Given, $\vec{a}=\hat{i}-\hat{j}-3\hat{k}$, $\vec{b}=4\hat{i}-3\hat{j}+\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+2\hat{k}$

Now, $\vec{b}+\vec{c}=(4\hat{i}-3\hat{j}+\hat{k})+(2\hat{i}-\hat{j}+2\hat{k})=6\hat{i}-4\hat{j}+3\hat{k}$

$$\therefore \quad \vec{a}\times(\vec{b}+\vec{c})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & -3\\ 6 & -4 & 3\end{vmatrix}=-15\hat{i}-21\hat{j}+2\hat{k} \qquad \text{...(i)}$$

$$\text{Also, } \vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & -3\\ 4 & -3 & 1\end{vmatrix}=-10\hat{i}-13\hat{j}+\hat{k}$$

$$\text{and } \vec{a}\times\vec{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & -3\\ 2 & -1 & 2\end{vmatrix}=-5\hat{i}-8\hat{j}+\hat{k}$$

$$\therefore \quad \vec{a}\times\vec{b}+\vec{a}\times\vec{c}=(-10\hat{i}-13\hat{j}+\hat{k})+(-5\hat{i}-8\hat{j}+\hat{k})$$

$$=-15\hat{i}-21\hat{j}+2\hat{k} \qquad \text{...(ii)}$$

From (i) and (ii), we have

$\vec{a}\times(\vec{b}+\vec{c})=\vec{a}\times\vec{b}+\vec{a}\times\vec{c}$

34. Given, position vector of $A=(\hat{i}+2\hat{j}+3\hat{k})$,

Position vector of $B=(2\hat{i}-\hat{j}+4\hat{k})$ and

Position vector of $C=(4\hat{i}+5\hat{j}-\hat{k})$

$$\therefore \quad \overrightarrow{AB}=(2\hat{i}-\hat{j}+4\hat{k})-(\hat{i}+2\hat{j}+3\hat{k})=\hat{i}-3\hat{j}+\hat{k}$$

$$\overrightarrow{AC}=(4\hat{i}+5\hat{j}-\hat{k})-(\hat{i}+2\hat{j}+3\hat{k})=3\hat{i}+3\hat{j}-4\hat{k}$$

$$\text{Now, } \overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -3 & 1\\ 3 & 3 & -4\end{vmatrix}$$

$$=(12-3)\hat{i}-(-4-3)\hat{j}+(3+9)\hat{k}=9\hat{i}+7\hat{j}+12\hat{k}$$

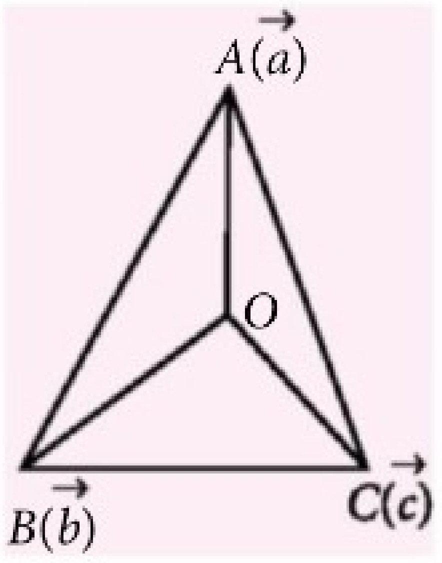
$$\therefore \quad \text{Area of } \Delta ABC=\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|$$

$$=\frac{1}{2}\left|(9\hat{i}+7\hat{j}+12\hat{k})\right|=\frac{1}{2}\cdot\sqrt{(9)^2+(7)^2+(12)^2}$$

$$=\frac{1}{2}\sqrt{274} \text{ sq. units}$$

35. *Refer to answer 85, Page no. 258 of MTG CBSE Champion Mathematics, Class-12*

OR



Let $OABC$ be a tetrahedron. Taking O as the origin, let the position vectors of the vertices A, B, C be $\vec{a},\vec{b}$ and $\vec{c}$ respectively. Let G, G_1, G_2, G_3 be the centroids of the faces ABC, OAB, OBC and OCA respectively. Then, Position vector of G, G_1, G_2, G_3 are

$\dfrac{\vec{a}+\vec{b}+\vec{c}}{3}$, $\dfrac{\vec{a}+\vec{b}}{3}$, $\dfrac{\vec{b}+\vec{c}}{3}$ and $\dfrac{\vec{c}+\vec{a}}{3}$ respectively.

Now, P.V. of a point dividing OG in the ratio 3 : 1

$$=\frac{3\left(\dfrac{\vec{a}+\vec{b}+\vec{c}}{3}\right)+1\cdot\vec{0}}{3+1}=\frac{\vec{a}+\vec{b}+\vec{c}}{4}$$

P.V. of a point dividing AG_2 in the ratio 3 : 1

$$=\frac{3\left(\dfrac{\vec{b}+\vec{c}}{3}\right)+1\cdot\vec{a}}{3+1}=\frac{\vec{a}+\vec{b}+\vec{c}}{4}$$

P.V. of a point dividing BG_3 in the ratio 3 : 1

$$=\frac{3\left(\dfrac{\vec{c}+\vec{a}}{3}\right)+1\cdot\vec{b}}{3+1}=\frac{\vec{a}+\vec{b}+\vec{c}}{4}$$

P.V. of a point dividing CG_1 in the ratio 3 : 1

$$=\frac{3\left(\dfrac{\vec{a}+\vec{b}}{3}\right)+1\cdot\vec{c}}{3+1}=\frac{\vec{a}+\vec{b}+\vec{c}}{4}$$

Thus, the point having position vector $\dfrac{\vec{a}+\vec{b}+\vec{c}}{4}$ is common to OG, AG_2, BG_3 and CG_1. Hence, the line joining the vertices of a tetrahedron to the centroids of opposite faces are concurrent.

36. *Refer to answer 118, Page no. 262 of MTG CBSE Champion Mathematics, Class-12*

◆◆

MONTHLY TEST DRIVE

XII

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 6 : Indefinite Integration

Time Taken : 60 Min.

Only One Option Correct Type

1. The value of $\int \frac{1}{x^2(x^4+1)^{3/4}}dx$, is

(a) $\left(1+\frac{1}{x^4}\right)^{1/4}+C$ (b) $-\left(1+\frac{1}{x^4}\right)^{1/4}+C$

(c) $-\frac{1}{4}\left(1+\frac{1}{x^4}\right)^{1/4}+C$ (d) None of these

2. $\int \frac{2+x^{-2}}{(x^{-1/2}+x^{3/2})^2}dx =$

(a) $\frac{1}{2}\log(1+x^2)-\frac{1}{x^2+1}+k$

(b) $\frac{1}{2}\left[\log\left(\frac{x^2}{x^2+1}\right)-\frac{1}{x^2+1}\right]+k$

(c) $\log(x^{-1/2}+x^{3/2})+k$ (d) $-\frac{1}{(x^{-1/2}+x^{3/2})}+k$

3. Let $I=\int \frac{e^x}{e^{4x}+e^{2x}+1}dx$, $J=\int \frac{e^{-x}}{e^{-4x}+e^{-2x}+1}dx$,

Then, for an arbitrary constant C, the value of $J - I$ equals

(a) $\frac{1}{2}\log\left(\frac{e^{4x}-e^{2x}+1}{e^{4x}+e^{2x}+1}\right)+C$

(b) $\frac{1}{2}\log\left(\frac{e^{2x}+e^{x}+1}{e^{2x}-e^{x}+1}\right)+C$

(c) $\frac{1}{2}\log\left(\frac{e^{2x}-e^{x}+1}{e^{2x}+e^{x}+1}\right)+C$

(d) $\frac{1}{2}\log\left(\frac{e^{4x}+e^{2x}+1}{e^{4x}-e^{2x}+1}\right)+C$

4. $\int\{\sqrt{\tan x}+\sqrt{\cot x}\}dx$ is equal to

(a) $\sin^{-1}(\sin x - \cos x) + C$

(b) $\sqrt{2}\sin^{-1}(\sin x - \cos x) + C$

(c) $\sqrt{2}\cos^{-1}(\sin x - \cos x) + C$

(d) None of these

5. $\int e^{\tan^{-1}x}(1+x+x^2)\, d(\cot^{-1}x)$ is equal to

(a) $-e^{\tan^{-1}x}+c$ (b) $e^{\tan^{-1}x}+c$

(c) $-x\,e^{\tan^{-1}x}+c$ (d) $x\,e^{\tan^{-1}x}+c$

6. If $\int \frac{1}{\cos^3 x\sqrt{2\sin 2x}}dx = (\tan x)^A + C(\tan x)^B + k$, where k is a constant of integration, then $A + B + C$ equals

(a) $\frac{16}{5}$ (b) $\frac{21}{5}$ (c) $\frac{7}{10}$ (d) $\frac{27}{10}$

One or More Than One Option(s) Correct Type

7. If $\int \frac{x^4+1}{x^6+1}dx=\tan^{-1}f(x)-\frac{2}{3}\tan^{-1}g(x)+C$, then

(a) both $f(x)$ and $g(x)$ are odd functions

(b) $f(x)$ is monotonic function

(c) $f(x) = g(x)$ has no real roots

(d) $\int \frac{f(x)}{g(x)}dx=-\frac{1}{x}+\frac{1}{3x^3}+c$

8. If $\int \frac{xe^x}{\sqrt{(1+e^x)}}dx=f(x)\sqrt{(1+e^x)}-2\ln g(x)+c$, then

(a) $f(x) = x - 1$ (b) $g(x)=\frac{\sqrt{(1+e^x)}-1}{\sqrt{(1+e^x)}+1}$

(c) $g(x)=\frac{\sqrt{(1+e^x)}+1}{\sqrt{(1+e^x)}-1}$ (d) $f(x) = 2(x - 2)$

9. If the integral $\int \frac{dx}{(1+x)^{1/2}+(1+x)^{2/3}}$ equals $k_1(1+x)^{1/3}+k_2(1+x)^{1/6}+k_3\log(1+(1+x)^{1/6})+C$, then

(a) $k_1 = k_2 = k_3$

(b) $2k_1 = k_3$

(c) k_2 and k_3 are of opposite sign

(d) None of these

10. $\int \frac{(\sqrt{1-x^2}+x)}{\sqrt{1-x^2}(1+x\sqrt{1-x^2})}\,dx$ equals

(a) $\frac{1}{\sqrt{3}}\log\left(\frac{\sqrt{3}+x-\sqrt{1-x^2}}{\sqrt{3}+\sqrt{1-x^2}-x}\right)+C$

(b) $\frac{1}{3}\log\left(\frac{\sqrt{3}+x-\sqrt{1-x^2}}{\sqrt{3}+\sqrt{1-x^2}-x}\right)+C$

(c) $-\frac{1}{3}\log\left(\frac{\sqrt{3}+x-\sqrt{1-x^2}}{\sqrt{3}+\sqrt{1-x^2}-x}\right)+C$

(d) $-\frac{1}{\sqrt{3}}\log\left(\frac{\sqrt{3}+\sqrt{1-x^2}-x}{\sqrt{3}+x-\sqrt{1-x^2}}\right)+C$

11. A curve $g(x) = \int x^{27}(1+x+x^2)^6(6x^2+5x+4)\,dx$ is passing through origin. Then

(a) $g(1) = \frac{3^7}{7}$

(b) $g(1) = \frac{2^7}{7}$

(c) $g(-1) = \frac{1}{7}$

(d) $g(-1) = \frac{3^7}{14}$

TELLING NUMBERS

Education Quality Index : how students did in maths, languages

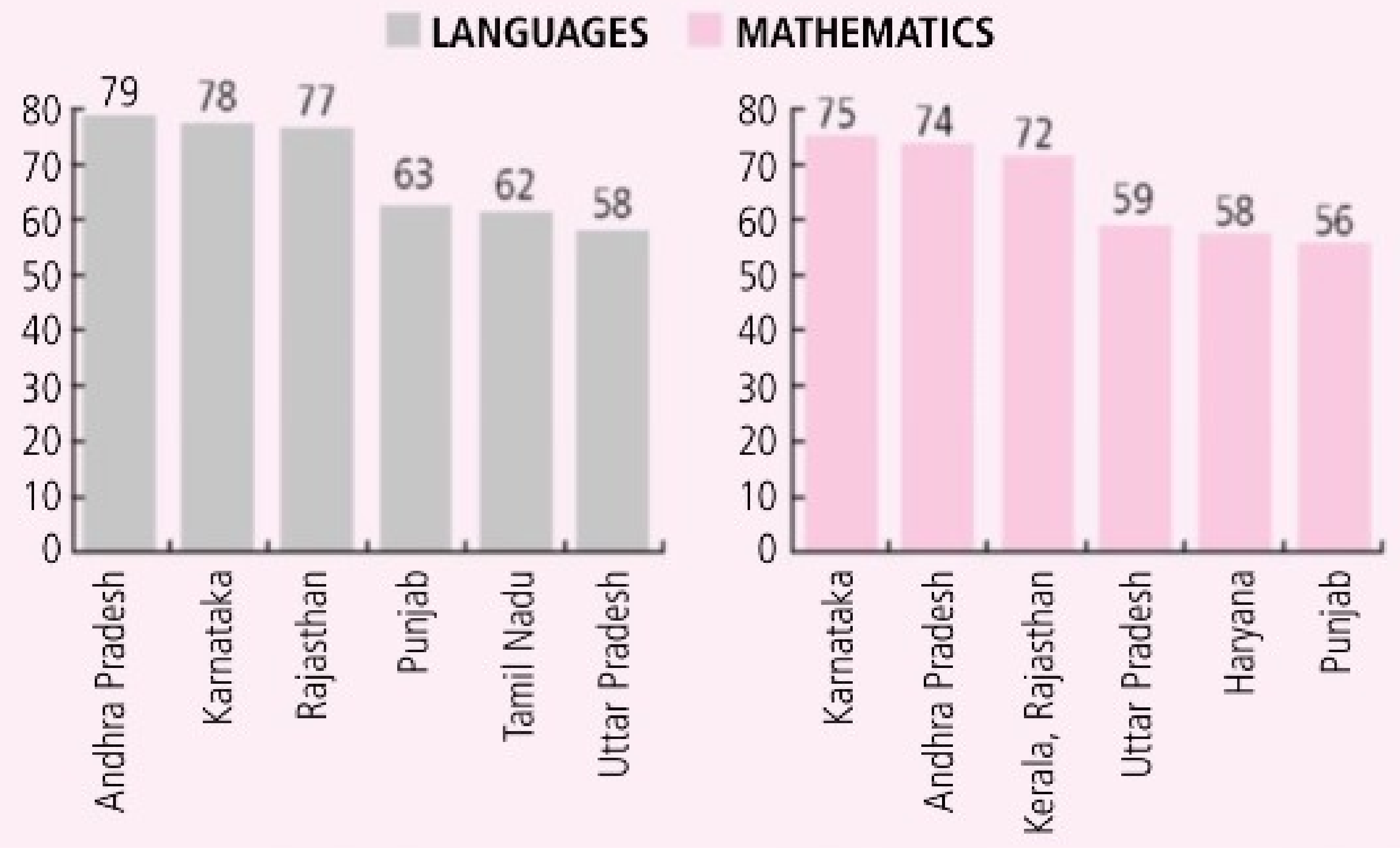
The graphs below show average scores by students of three classes in 20 "large states" in the National Achievement Survey 2017, and detailed in the NITI Aayog's report on the School Education Quality Index 2019, released on Monday. Performance in languages and mathematics was among various indicators chosen to assess the "overall effectiveness, quality and efficiency of the Indian school education system", and came under the domain of "learning outcomes". The data were sourced mostly from publicly available sources.

As reported on Tuesday, Kerala had the best overall performance at 76.6 percent and Uttar Pradesh had the worst performance, with 36.4 percent.

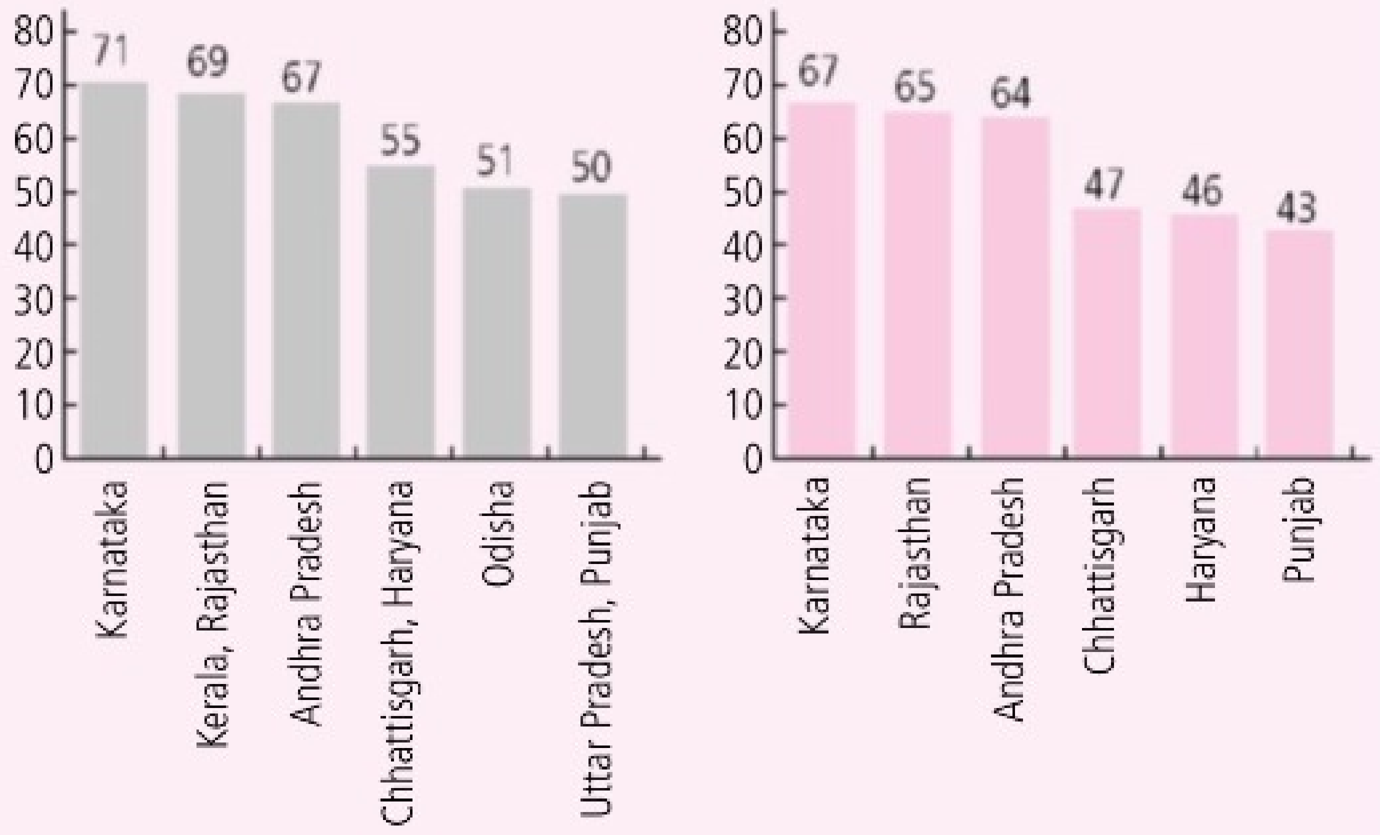
In Class 3, Andhra Pradesh and Karnataka have the highest average scores in language and mathematics, while Uttar Pradesh and Punjab have the lowest. For class 5, Karnataka tops the list with the highest scores in both language and mathematics, while Uttar Pradesh and Punjab are again at the bottom. For class 8, Rajasthan has the highest average scores for both language and mathematics, whereas Telangana, Uttar Pradesh and Punjab have the lowest scores.

TOP 3 & BOTTOM 3 (LARGE STATES)

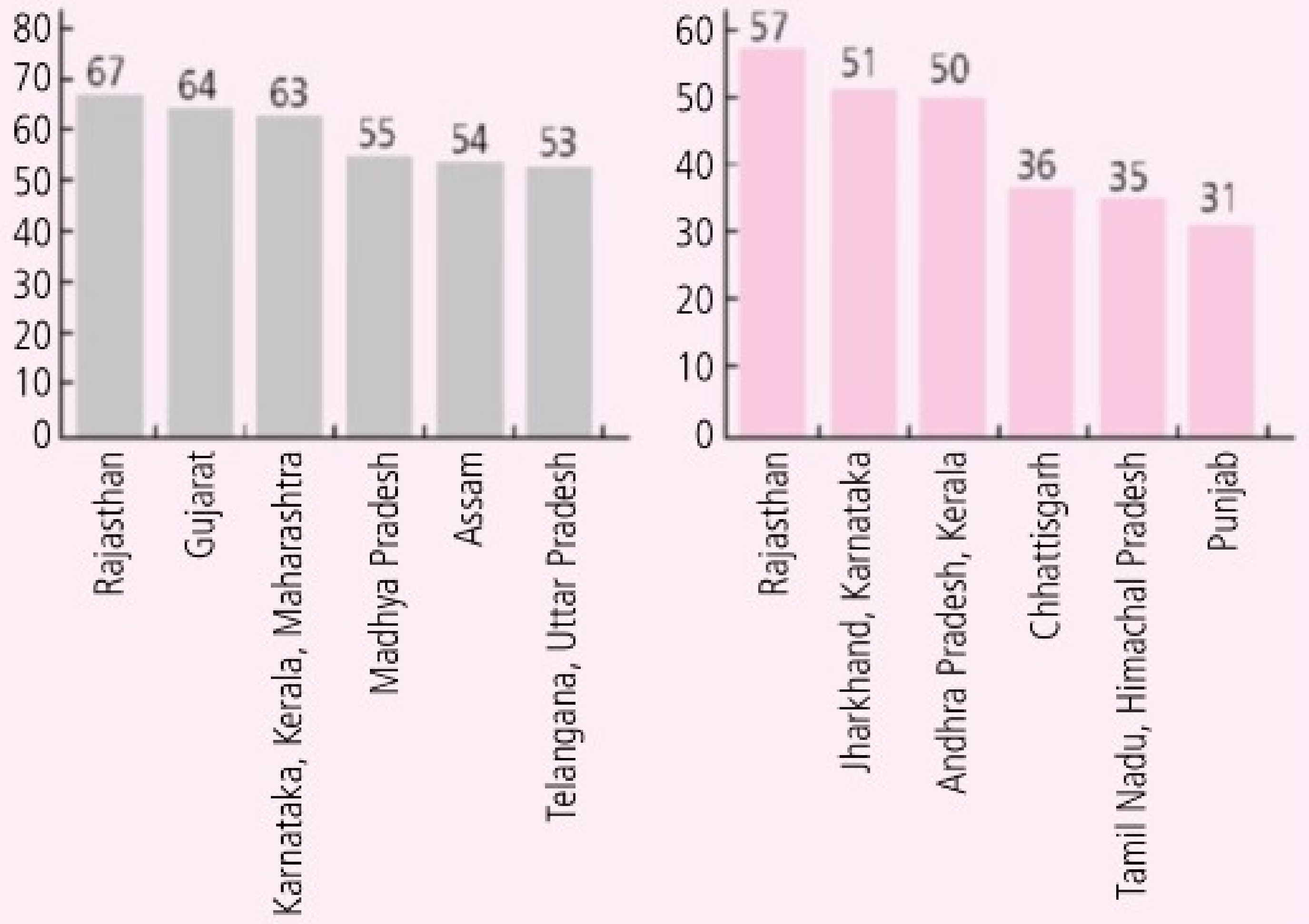
CLASS 3



CLASS 5



CLASS 8



Courtesy : The Indian Express

12. If $\int \operatorname{cosec} 2x\, dx = f\{g(x)\} + c$, then
(a) range of $g(x) = (-\infty, \infty)$
(b) domain of $f(x) = (-\infty, \infty) - \{0\}$
(c) $g'(x) = \sec^2 x$
(d) $f'(x) = 1/x$ for all $x \in (0, \infty)$

13. If $\int \frac{\cos 4x + 1}{\cot x - \tan x} dx = Af(x) + B$, then
(a) $A = -\frac{1}{8}$
(b) $B = \frac{1}{2}$
(c) $f(x)$ has fundamental period $\frac{\pi}{2}$
(d) None of these

Comprehension Type

Integrals of the form $\int R(x, \sqrt{ax^2 + bx + c})$ are calculated with the aid of one of the three Euler substitutions

(i) $\sqrt{ax^2 + bx + c} = t \pm x\sqrt{a}$ if $a > 0$

(ii) $\sqrt{ax^2 + bx + c} = tx \pm \sqrt{c}$ if $c > 0$

(iii) $\sqrt{ax^2 + bx + c} = (x - a)t$ if $ax^2 + bx + c = a(x - \alpha)(x - \beta)$ *i.e.*, if α is a real root of $ax^2 + bx + c = 0$

14. Which of the following functions does not appear in the primitive of $\int \frac{dx}{x + \sqrt{x^2 - x + 1}}$ if t is a function of x?
(a) $\log_e |t|$
(b) $\log_e |t - 2|$
(c) $\log_e |t - 1|$
(d) $\log_e |t + 1|$

15. $\int \frac{x\,dx}{(\sqrt{7x - 10 - x^2})^3}$ can be evaluated for
(a) $x = \frac{5 + 2t^2}{t^2 + 1}$
(b) $x = \frac{5 - t^2}{t^2 + 2}$
(c) $x = \frac{2t^2 - 5}{3t^2 - 1}$
(d) None of these

Matrix Match Type

16. Match the columns:

Column-I		Column-II	
P.	$\int (\tan x)^{1/3} dx = A \ln \frac{(t^4 - t^2 + 1)}{(t^2 + 1)^2} + B \tan^{-1}\left(\frac{2t^2 - 1}{\sqrt{3}}\right) + c,$ where $t = \tan^{1/3} x$	1.	$A = \frac{1}{4}$, $B = \frac{\sqrt{3}}{2}$
Q.	$\int \left(\frac{\sin x + \sin^3 x}{\cos 2x}\right) dx = A \cos x + B \ln \left\|\frac{\sqrt{2}\cos x + 1}{\sqrt{2}\cos x - 1}\right\| + C$	2.	$A = \frac{1}{3}$, $B = -\frac{1}{6}$
R.	$\int \frac{dx}{(x^2 + 1)(x^2 + 4)} = A \tan^{-1} x + B \tan^{-1}\left(\frac{x}{2}\right) + c$	3.	$A = \frac{1}{2}$, $B = \frac{3}{4\sqrt{2}}$

	P	Q	R
(a)	2	3	1
(b)	1	3	2
(c)	3	2	1
(d)	2	1	3

Numerical Value Type

17. Let $f(x)$ be a quadratic function such that $f(0) = 1$ and $f(-1) = 4$, if $\int \frac{f(x)}{x^2 (1 + x)^2} dx$ is a rational function, then the value of $f(10)$ is

18. Let $f(x) = \int x^{\sin x}(1 + x \cos x . \ln x + \sin x) dx$ and $f\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4}$. Then the value of $|\cos(f(\pi))|$ is

19. If $\int \sqrt{\frac{\cos^3 x}{\sin^{11} x}} dx = -2\{A \tan^{-9/2} x + B \tan^{-5/2} x\} + C$, then $A + B$ is equal to

20. If $\int \left(\frac{\cos 8x - \cos 7x}{1 + 2\cos 5x}\right) dx = \frac{\sin 3x}{a} - \frac{\sin 2x}{b} + c$, then the value of $(a^b)^4$ must be

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CONCEPT BOOSTER

Straight Lines

Class XI

*ALOK KUMAR

GENERAL EQUATION OF A STRAIGHT LINE

Any equation of first degree of the form $ax + by + c = 0$, where a, b, c, are constants always represents a straight line (at least one out of a and b is non-zero).

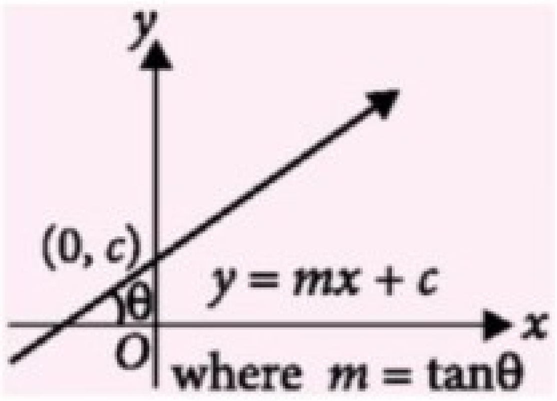
- **Slope or gradient of a line:** If θ is the angle at which a straight line is inclined to the positive direction of x-axis, then $\tan\theta$ is called the slope or gradient of the line, where $0 \le \theta < 180°$ $(\theta \ne 90°)$.
- If a line is parallel to x-axis, then its slope is $\tan 0° = 0$.
- Slope of a line perpendicular to x-axis is not defined.
- Slope of the line passing through two points (x_1, y_1) and (x_2, y_2) is given by $\dfrac{y_2 - y_1}{x_2 - x_1}$.

INTERCEPTS OF A STRAIGHT LINE ON THE AXES

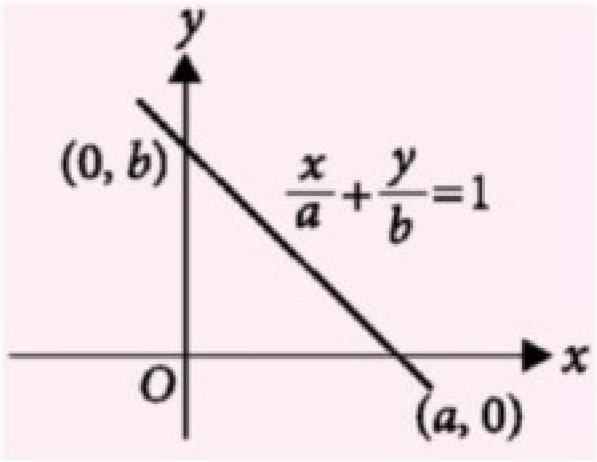
- **Intercept of a line on x-axis :** If a line cut the x-axis at $(a, 0)$, then 'a' is called the intercept of a line on x-axis or the x-intercept. If a line intersect the x-axis at $(-2, 0)$, then the length of intercept is $|-2| = 2$.
- **Intercept of a line on y-axis:** If a line cuts y-axis at $(0, b)$, then b is called the intercept of a line on y-axis or y-intercept.

EQUATION OF A LINE IN DIFFERENT FORMS

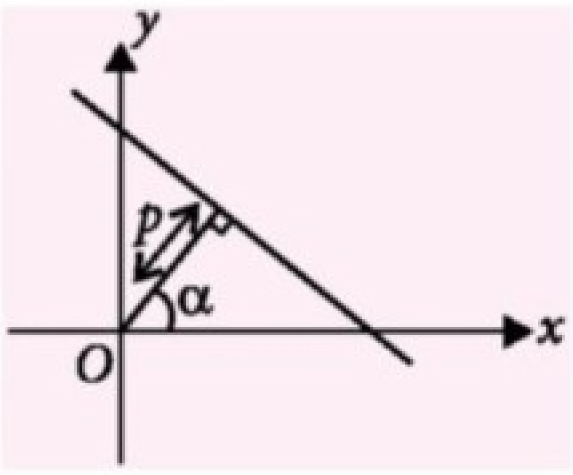
- **Point-slope form :** The equation of a line passing through the point (x_1, y_1) and having slope m is given by $(y - y_1) = m(x - x_1)$.
- **Two point form:** The equation of a straight line passing through two given points (x_1, y_1) and (x_2, y_2) is given by $(y - y_1) = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
- **Slope intercept form:** Equation of a line whose slope is $m(m = \tan\theta)$ and which cut an intercept c on the y-axis (*i.e.*, which passes through the point $(0, c)$) is given by $y = mx + c$.



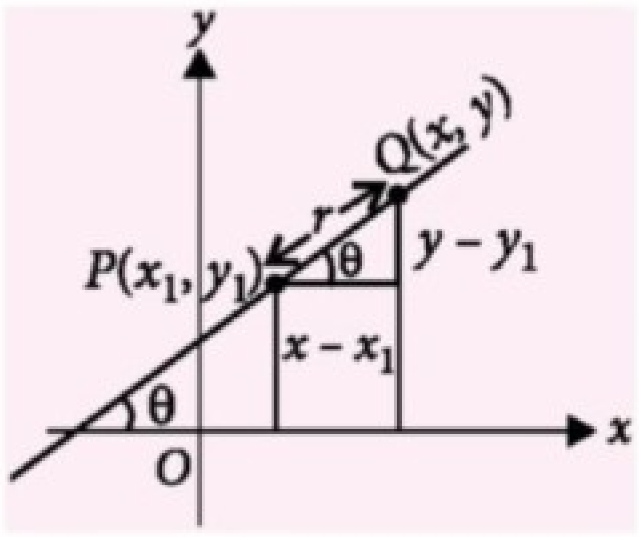
- **Intercept form:** The equation of the line which cuts off intercepts a and b on x-axis and y-axis respectively is given by $\dfrac{x}{a} + \dfrac{y}{b} = 1$.



- **Normal form :** The equation of a straight line upon which the length of perpendicular from the origin is p and the perpendicular makes an angle α with the positive direction of x-axis is given by $x\cos\alpha + y\sin\alpha = p$. In normal form of equation of a straight line p is always taken as positive and α is measured from positive direction of x-axis in anticlockwise direction between 0 and 2π.



- **Parametric form :** The equation of line passing through a given point $P(x_1, y_1)$ and making an angle θ with the positive direction of x-axis is given by $\dfrac{x - x_1}{\cos\theta} = \dfrac{y - y_1}{\sin\theta} = r$,



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where r is the distance between the variable point $Q(x, y)$ and the fixed point $P(x_1, y_1)$.

- Any point on the line will be of the form $(x_1 + r\cos\theta, y_1 + r\sin\theta)$. For different values of r, we will get different points on the line.
- If $P(x_1, y_1)$ is any point on the line which makes an angle θ with the positive direction of x axis, then there will be two points on the line at a distance r from $P(x_1, y_1)$. One will be relatively upward if r is taken positive and other will be relatively downward if r is taken negative.

POSITION OF A POINT WITH RESPECT TO A GIVEN LINE

Let the given line be $ax + by + c = 0$, and we have to check whether the point (α, β) lies above the line or below the line. To check first draw a perpendicular line from $P(\alpha, \beta)$ on the x-axis which intersect the given line at Q. x-coordinate of Q will be α and corresponding to α, y-coordinate of Q will be $\beta' = -\left(\dfrac{a\alpha + c}{b}\right)$. Now

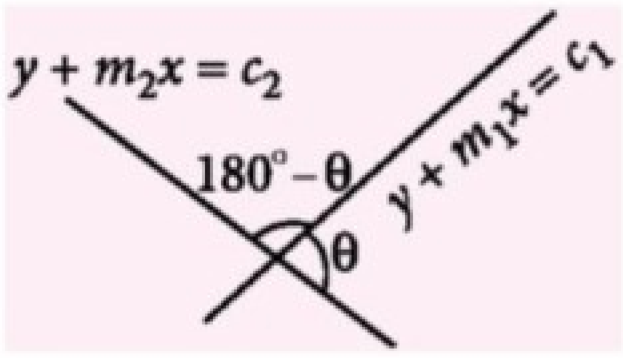
if $\beta > \beta'$, then the point will lie above the line
if $\beta = \beta'$, then the point will lie on the line
and if $\beta < \beta'$, then the point will lie below the line.

POSITION OF TWO POINTS WITH RESPECT TO A LINE

Let the given line be $ax + by + c = 0$ and $P(x_1, y_1)$, $Q(x_2, y_2)$ be two given points.

Now if $\dfrac{ax_1 + by_1 + c}{ax_2 + by_2 + c} < 0$, then both $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ are of opposite sign, then the points P and Q will lie on the opposite side of the line $ax + by + c = 0$ and, if $\dfrac{ax_1 + by_1 + c}{ax_2 + by_2 + c} > 0$, then both $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ are of the same sign, then the points P and Q will lie on the same side of the given line.

ANGLE BETWEEN TWO STRAIGHT LINES

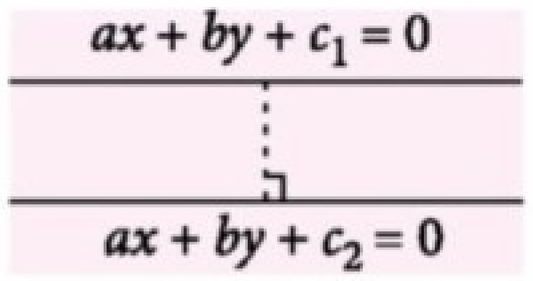


- If θ is the acute angle between two lines, then $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1m_2}\right|$, where m_1 and m_2 are the slopes of the two lines and are finite.
- If the two lines are perpendicular to each other, then $m_1m_2 = -1$.
- Any line perpendicular to $ax + by + c = 0$ is of the form $bx - ay + k = 0$.
- If the two lines are parallel or are coincident, then $m_1 = m_2$.
- Any line parallel to $ax + by + c = 0$ is of the form $ax + by + k = 0$.
- If any of the two lines is perpendicular to x-axis, then the slope of that line is not define (infinite).
- Let $m_1 = \infty$, then

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right| = \left|\frac{1 - \dfrac{m_2}{m_1}}{\dfrac{1}{m_1} + m_2}\right| = \left|\frac{1}{m_2}\right|$$

or $\theta = |90° - \alpha|$, where $\tan\alpha = m_2$ *i.e.*, angle θ is the complimentary to the angle which the oblique line makes with the x-axis.

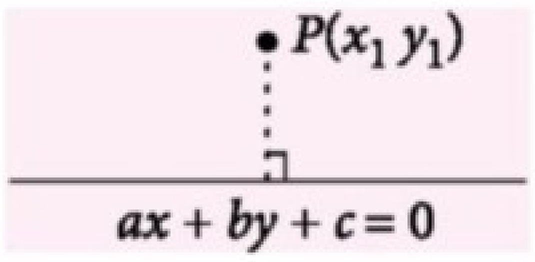
DISTANCE BETWEEN TWO PARALLEL LINES



The distance between two parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is

$$\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

PERPENDICULAR DISTANCE OF A POINT FROM A LINE



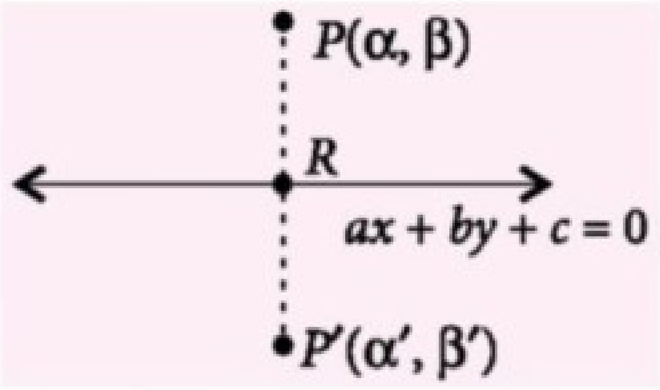
The length of the perpendicular from $P(x_1, y_1)$ on $ax + by + c = 0$ is

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

IMAGE OF A POINT W.R.T. A LINE

Let the given line be $ax + by + c = 0$ and we have to find the image of the point $P(\alpha, \beta)$.

- Slope of PP' (where P' is image of P) will be b/a, because PP' is perpendicular to given line. Hence equation of PP' is $y - \beta = b/a(x - \alpha)$



- Let the point of intersection of line PP' with the given line be $R(p, q)$. Clearly, R is the mid point of PP'.

$$\therefore\ \frac{\alpha + \alpha'}{2} = p \Rightarrow \alpha' = 2p - \alpha$$

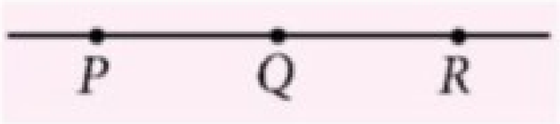
$$\text{Also, } \frac{\beta + \beta'}{2} = q \ \Rightarrow\ \beta' = 2q - \beta$$

Hence the image is $(2p - \alpha, 2q - \beta)$.

COLLINEARITY OF THREE POINTS

- Three points $P(x_1, y_1)$, $Q(x_2, y_2)$, $R(x_3, y_3)$ are collinear, if area of triangle formed by these three points should be equal to zero

  $$i.e., \ \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$



- Any one point of the given three points should divide other two points either internally or externally.

  $$\Rightarrow x_3 = \frac{mx_2 + nx_1}{m+n} \text{ and } y_3 = \frac{my_2 + ny_1}{m+n} \text{ for some}$$

  real m and n.

- Slope of PQ = Slope of QR $\Rightarrow \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{y_3 - y_2}{x_3 - x_2}$

CONCURRENCY OF THREE STRAIGHT LINES

The condition for 3 lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3 = 0$ to be concurrent is $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$.

BISECTORS OF ANGLE BETWEEN TWO GIVEN LINES

Let $a_1x + b_1y + c_1 = 0$ ...(i)

and $a_2x + b_2y + c_2 = 0$ ...(ii) are two intersecting lines. Let $P(x, y)$ be any point on either of the two bisectors of angles of (i) and (ii). Then P is equidistance from (i) and (ii).

$$\Rightarrow \left|\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}}\right| = \left|\frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}\right|$$

$$\Rightarrow \frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm\frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}} \quad \text{...(iii)}$$

which are the required equations of the two bisectors of angles between (i) and (ii).

If the two given lines are not perpendicular *i.e.*, $a_1a_2 + b_1b_2 \neq 0$, then one of the equations given by (iii) is the equation of the bisector of acute angle and the other that of the obtuse angle.

- Equation of acute and obtuse angle bisectors:

  Take one of the given lines and let its slope be m_1 and take one of the bisectors and let its slope be m_2. If θ be the acute angle between them, find

  $$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|.$$

  (i) If $\tan\theta > 1$, then the bisector taken is the bisector of the obtuse angle and the other one will be the bisector of the acute angle.

  (ii) If $\tan\theta < 1$, then the bisector taken is the bisector of the acute angle and the other one will be the bisector of the obtuse angle.

- If the constant term c_1 and c_2 in the two equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are of the same sign.

  Case-(i) : If $a_1a_2 + b_1b_2 > 0$, then

  $$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

  will give the equation of obtuse angle bisector and

  $$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = -\frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

  will give the equation of acute angle bisector.

  Case (ii): If $a_1a_2 + b_1b_2 < 0$, then

  $$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

  will give the equation of acute angle bisector and $\dfrac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = -\dfrac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$ will give the equation of obtuse angle bisector.

  Note : Whether both the lines are perpendicular or not but the angle bisectors of these lines will always be mutually perpendicular.

- The equation of the bisector of the angle which contain a given point:

  The equation of the bisector of the angle between the two lines containing the point (α, β) is

  $$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

  if $a_1\alpha + b_1\beta + c_1$ and $a_2\alpha + b_2\beta + c_2$ are of the same signs or $\dfrac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = -\dfrac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$

  if $a_1\alpha + b_1\beta + c_1$ and $a_2\alpha + b_2\beta + c_2$ are of the opposite signs

- The equation of the bisector of the angle containing the origin:

  Write the equations of the two lines so that the constants c_1 and c_2 are positive. Then the equation $\dfrac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \dfrac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$ is the equation of the bisector containing the origin.

If $a_1a_2 + b_1b_2 < 0$, then the origin will lie in the acute angle and if $a_1a_2 + b_1b_2 > 0$, then origin will lie in the obtuse angle.

FAMILY OF LINES

Suppose $L_1 = a_1x + b_1y + c_1 = 0$ and $L_2 = a_2x + b_2y + c_2 = 0$ are two intersecting lines and let the point of their intersection be (α, β). Now if we write these two equations in this form $L_1 + \lambda L_2 = 0$ (where λ is a parameter)

i.e., $(a_1x + b_1y + c_1) + \lambda(a_2x + b_2y + c_2) = 0$ ...(i)

then for different values of λ, (i) will give different straight lines.

Hence (i) represent a family of straight lines passing through the point of intersection of

$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$.

- Family of lines perpendicular to a given line $ax + by + c = 0$ is given by $bx - ay + k = 0$, where k is a parameter.
- Family of lines parallel to a given line $ax + by + c = 0$ is given by $ax + by + k = 0$, where k is a parameter.

PAIR OF STRAIGHT LINES

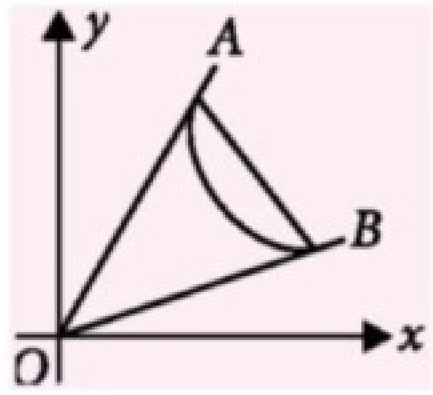
- The general equation of second degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, represents a pair of straight lines if, $\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$ and $h^2 \geq ab$.

  $\Rightarrow$ $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ and $h^2 \geq ab$.
- A homogeneous equation of second degree in x and y, *i.e.*, $ax^2 + 2hxy + by^2 = 0$ represents a pair of straight lines passing through the origin if $h^2 \geq ab$.
- If the lines through the origin whose joint equation is $ax^2 + 2hxy + by^2 = 0$, are $y = m_1x$ and $y = m_2x$, then $y^2 - (m_1 + m_2)xy + m_1m_2x^2 = 0$ and $y^2 + \frac{2h}{b}xy + \frac{a}{b}x^2 = 0$ are identical, so that $m_1 + m_2 = -\frac{2h}{b}$, $m_1m_2 = \frac{a}{b}$.
- If θ be the angle between two lines, through the origin, then

  $$\tan\theta = \pm\frac{\sqrt{(m_1+m_2)^2 - 4m_1m_2}}{1+m_1m_2} = \pm\frac{2\sqrt{h^2-ab}}{a+b}$$

  The lines are perpendicular if $a + b = 0$ and coincident if $h^2 = ab$.
- Joint Equation of Pair of Lines Joining the Origin and the Points of Intersection of a Curve and a Line:

If the line $lx + my + n = 0$, ($(n \neq 0)$ *i.e.*, the line does not pass through origin) cut the curve $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ at two points A and B, then the joint equation of straight lines passing through A and B and the origin is given by homogenizing the equation of the curve by the equation of the line *i.e.*, $ax^2 + 2hxy + by^2 + (2gx + 2fy)$

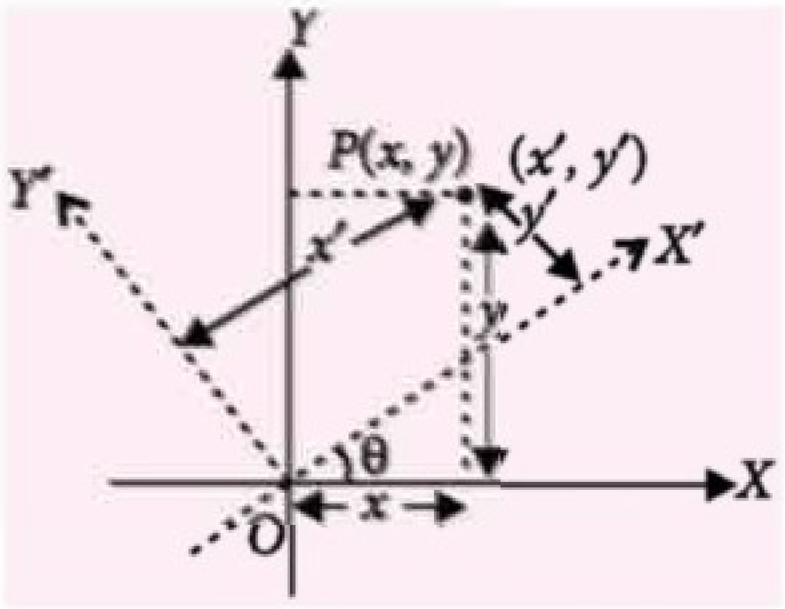


$$\left(\frac{lx+my}{-n}\right) + c\left(\frac{lx+my}{-n}\right)^2 = 0$$

is the equation of the lines OA and OB.

ROTATION OF CO-ORDINATE AXES

Let OX, OY be the original axes and OX' and OY' be the new axes obtained after rotating OX and OY through an angle θ in the anticlockwise direction. Let P be any point in the plane having coordinates (x, y) with respect to axes OX and OY and (x', y') with respect to axes OX' and OY'.



Then $x = x' \cos\theta - y' \sin\theta$, $y = x' \sin\theta + y' \cos\theta$ ...(i)

and $x' = x \cos\theta + y \sin\theta$, $y' = -x \sin\theta + y \cos\theta$. ...(ii)

The above transformation can also be displaced by a table.

	x'	y'
x	$\cos\theta$	$-\sin\theta$
y	$\sin\theta$	$\cos\theta$

If $f(x, y) = 0$ is the equation of a curve then it's transformed equation is $f(x'\cos\theta - y'\sin\theta, x'\sin\theta + y'\cos\theta) = 0$.

PROBLEMS

Single Correct Answer Type

1. Incentre of the triangle formed by the lines $y = x$, $y = 3x$ and $y = 8 - 3x$ is

(a) $\left(\frac{4}{5}, \frac{2-\sqrt{5}}{3}\right)$ (b) $\left(\frac{4}{5}, \frac{2+\sqrt{5}}{3}\right)$

(c) $\left(\frac{4}{3}, \frac{2+2\sqrt{5}}{3}\right)$ (d) $\left(\frac{4}{3}, \frac{2-2\sqrt{5}}{3}\right)$

2. Consider the triangle OAB, where $O = (0, 0)$, $B = (3, 4)$. If the orthocentre of a triangle is $H(1, 4)$, then coordinate of A is

(a) $(0, 15/4)$ (b) $(0, 17/4)$
(c) $(0, 21/4)$ (d) $(0, 19/4)$

3. The diagonals of the parallelogram whose sides are $lx + my + n = 0$, $lx + my + n' = 0$, $mx + ly + n = 0$, $mx + ly + n' = 0$ include an angle

(a) $\pi/2$ (b) $\pi/3$
(c) $\tan^{-1}\left(\frac{l^2 - m^2}{l^2 + m^2}\right)$ (d) $\tan^{-1}\left(\frac{2lm}{l^2 + m^2}\right)$

4. Equation $ax^2 + 2hxy + by^2 = 0$ represents a pair of lines, combined equation of lines that can be obtained by taking the mirror of lines about the x-axis is

(a) $ax^2 + 2hxy + by^2 = 0$ (b) $bx^2 + 2hxy + ay^2 = 0$
(c) $bx^2 - 2hxy + ay^2 = 0$ (d) none of these

5. A variable line drawn through the point $(1, 3)$ meets the x-axis at A and y-axis at B. If the rectangle $OAPB$ is completed, where 'O' is the origin, then locus of 'P' is

(a) $\frac{1}{y} + \frac{3}{x} = 1$ (b) $x + 3y = 1$
(c) $\frac{1}{x} + \frac{3}{y} = 1$ (d) $3x + y = 1$

6. The least value of $|\alpha|$, for which the lines $x = \alpha + m$, $y = -2$ and $y = mx$ are concurrent, is

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) 0 (d) 1

7. The straight lines $L_1 \equiv 4x - 3y + 2 = 0$, $L_2 \equiv 3x + 4y - 4 = 0$ and $L_3 \equiv x - 7y + 6 = 0$

(a) form a right angled triangle
(b) form a right angled isosceles triangle
(c) are concurrent (d) none of these

8. $P(x, y)$ is called a good point if $x, y \in N$. Total number of good points lying inside the quadrilateral formed by the lines $2x + y = 2$, $x = 0$, $y = 0$ and $x + y = 5$, is equal to

(a) 4 (b) 2 (c) 6 (d) 1

9. The sides of a rectangle are $x = 0$, $y = 0$, $x = 4$ and $y = 3$. The equation of straight line having slope $\frac{1}{2}$ that divides the rectangle into two equal halves is

(a) $2y = x + 1$ (b) $2x = y + 1$
(c) $2y + x = 1$ (d) $2x + y = 1$

10. The equation of the bisector of angle between the lines $x + y = 1$ and $7x - y = 3$ that contains the point $(2, 3)$ is

(a) $3x + y - 1 = 0$ (b) $x - 3y + 1 = 0$
(c) $2x + y - 1 = 0$ (d) $x - 2y + 1 = 0$

11. The locus of point of intersection of lines $x\cos\alpha + y\sin\alpha = a$ and $x\sin\alpha - y\cos\alpha = b$ is (α is a variable)

(a) $2(x^2 + y^2) = a^2 + b^2$ (b) $x^2 - y^2 = a^2 - b^2$
(c) $x^2 + y^2 = a^2 + b^2$ (d) None of these

12. If $x^2 - 7xy - y^2 = 0$, $x + y = 2$, represents the sides of a triangle, then distance between orthocenter and the side of triangle is

(a) 2 (b) $\sqrt{3}$ (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

13. The equation of an acute angle bisector between lines $(\sqrt{3} - 1)y = (\sqrt{3} + 1)x$ and $(\sqrt{3} + 1)y = (\sqrt{3} - 1)x$ is

(a) $y = x$ (b) $x + y = 0$
(c) $y = x + 1$ (d) $y = -x + 1$

14. If the line $y = \sqrt{3}x$ cuts the curve $x^3 + y^2 + 3x^2 + 8 = 0$ at the points A, B, C, then $OA \cdot OB \cdot OC$ (O being origin) equals

(a) -32 (b) -64 (c) 108 (d) -100

15. If one vertex of an equilateral triangle of side 2 is the origin and another vertex lies on the line $x = \sqrt{3}y$, then third vertex can be

(a) $(0, 2)$ (b) $(-\sqrt{3}, -1)$
(c) $(2, 0)$ (d) $(\sqrt{3}, 1)$

16. A line passing through the point $(2, 2)$ and the axes enclose an area λ. The intercepts on the axes made by the line are given by the two roots of

(a) $x^2 - 2|\lambda|\,x + |\lambda| = 0$ (b) $x^2 + |\lambda|\,x + 2\,|\lambda| = 0$
(c) $x^2 - |\lambda|\,x + 2\,|\lambda| = 0$ (d) none of these

17. Area of the triangle formed by the line $x + y = 3$ and angle bisector of the pair of straight lines $x^2 - y^2 + 2y = 1$ is

(a) 2 sq.units (b) 4 sq.units
(c) 6 sq.units (d) 8 sq.units

18. Let $P \equiv (-1, 0)$, $Q \equiv (0, 0)$ and $R \equiv (3, 3\sqrt{3})$ be three points. Then the equation of the bisector of the angle PQR is

(a) $\frac{\sqrt{3}}{2}x + y = 0$ (b) $x + \sqrt{3}y = 0$
(c) $\sqrt{3}x + y = 0$ (d) $x + \frac{\sqrt{3}}{2}y = 0$

19. The incentre of the triangle with vertices $(1, \sqrt{3})$, $(0, 0)$ and $(2, 0)$ is

(a) $\left(1, \frac{\sqrt{3}}{2}\right)$ (b) $\left(2, \frac{1}{\sqrt{3}}\right)$
(c) $\left(\frac{2}{3}, \frac{1}{\sqrt{3}}\right)$ (d) $\left(1, \frac{1}{\sqrt{3}}\right)$

20. If x_1, x_2, x_3 as well as y_1, y_2, y_3 are in G.P., with the same common ratio, then the points (x_1, y_1), (x_2, y_2) and (x_3, y_3)
(a) lie on a straight line (b) lie on an ellipse
(c) lie on a circle (d) are vertices of a triangle

21. Let PQR be a right angled isosceles triangle, right angled at $P(2, 1)$. If the equation of the line QR is $2x + y = 3$, then the equation representing the pair of lines PQ and PR is
(a) $3x^2 - 3y^2 + 8xy + 20x + 10y + 25 = 0$
(b) $3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$
(c) $3x^2 - 3y^2 + 8xy + 10x + 15y + 20 = 0$
(d) $3x^2 - 3y^2 - 8xy + 10x - 15y - 20 = 0$

22. If $P \equiv (1, 0)$, $Q \equiv (-1, 0)$ and $R \equiv (2, 0)$ are three given points, then locus of the point S satisfying the relation $SQ^2 + SR^2 = 2SP^2$ is
(a) a straight line parallel to x-axis
(b) a circle passing through the origin.
(c) a circle with the centre at the origin
(d) a straight line parallel to y-axis.

Multiple Correct Answer Type

23. Under rotation of axes through θ, $x\cos\alpha + y\sin\alpha = P$ changes to $X\cos\beta + Y\sin\beta = P$, then
(a) $\cos\beta = \cos(\alpha - \theta)$ (b) $\cos\alpha = \cos(\beta - \theta)$
(c) $\sin\beta = \sin(\alpha - \theta)$ (d) $\sin\alpha = \sin(\beta - \theta)$

24. The angle through which the co-ordinate axes be rotated so that xy-term in the equation $5x^2 + 4\sqrt{3}xy + 9y^2 = 0$ may be missing is
(a) $\frac{\pi}{6}$ (b) $\frac{-\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

25. Sides of a rhombus are parallel to the lines $x + y - 1 = 0$ and $7x - y - 5 = 0$. It is given that diagonals of the rhombus intersect at $(1, 3)$ and one vertex 'A' of the rhombus lies on the line $y = 2x$. Then the coordinates of the vertex A are
(a) $\left(\frac{8}{5}, \frac{16}{5}\right)$ (b) $\left(\frac{7}{15}, \frac{14}{15}\right)$
(c) $\left(\frac{6}{5}, \frac{12}{5}\right)$ (d) $\left(\frac{4}{15}, \frac{8}{15}\right)$

26. If the points $(-2, 1)$, $(3, 4)$ and (a^2, a) lie on the same side of the straight line $6x - 7y - 3 = 0$, then a can lie in
(a) $(0, 1)$ (b) $\left(\frac{6}{5}, \frac{7}{5}\right)$ (c) $(2, 4)$ (d) $(-1, 1)$

27. If $x^2 + 2hxy + y^2 = 0$ $(h \neq 1)$ represents the equations of the straight lines through the origin which make an angle α with the straight line $y + x = 0$, then (m_1 and m_2 are slopes of given pair of straight lines)
(a) $\sec 2\alpha = h$ (b) $\cos\alpha = \sqrt{\frac{1+h}{2h}}$
(c) $m_1 + m_2 = -2\sec 2\alpha$ (d) $\cot\alpha = \sqrt{\frac{h+1}{h-1}}$

Assertion & Reason Type

Directions : *In the following questions, Statement-1 is followed by Statement-2. Mark the correct choice as :*
(a) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true, Statement–2 is true, Statement-2 is not a correct explanation for Statement-1
(c) Statement-1 is true, Statement-2 is false
(d) Statement-1 is false, Statement-2 is true

28. Statement-1 : A chord $y = mx + c$ of the curve $3x^2 - y^2 - 2x + 4y = 0$, which passes through the point $(1, -2)$, subtend a right angle at the origin.
Statement-2 : Lines represented by the equation $(3c + 2m)x^2 - 2(1 + 2m)xy + (4 - c)y^2 = 0$ are perpendicular, if $c + m + 2 = 0$.

29. Statement-1 : If algebraic sum of perpendicular distances from $(-2, 0)$, $(3, 1)$ and $(4, 2)$ to the line $ax + by + c = 0$ is zero, then line must pass through $\left(\frac{5}{3}, 1\right)$.
Statement-2 : If algebraic sum of perpendicular distances from $A_i(x_i, y_i)$, $i = 1, 2, 3, \ldots n$ to $ax + by + c = 0$ is zero, then line must pass through centroid of polygon having vertices at (x_i, y_i).

30. Statement-1 : The vertices of a triangle are $A(x_1, x_1 \tan\theta_1)$, $B(x_2, x_2 \tan\theta_2)$ and $C(x_3, x_3 \tan\theta_3)$. If the circumcentre of the triangle ABC coincides with origin and orthocentre $H(x', y')$, then $\frac{y'}{x'} = \frac{\sin\theta_1 + \sin\theta_2 + \sin\theta_3}{\cos\theta_1 + \cos\theta_2 + \cos\theta_3}$.
Statement-2 : In a triangle circumcentre, centroid and orthocentre are collinear.

Comprehension Type

Paragraph for Q. No. 31 to 33

In a ΔPQR, with $PQ = r$, $QR = p$, $PR = q$, the cosine values of the angles are given by $\cos P = \frac{q^2 + r^2 - p^2}{2qr}$; $\cos Q = \frac{p^2 + r^2 - q^2}{2pr}$; $\cos R = \frac{p^2 + q^2 - r^2}{2pq}$, and the area of ΔPQR is $\Delta = \frac{1}{2}pq\sin R = \frac{1}{2}qr\sin P = \frac{1}{2}pr\sin Q$.

Let $ABCD$ be a parallelogram whose diagonal equations are $AC \equiv x + 2y - 3 = 0$; $BD \equiv 2x + y - 3 = 0$. If $AC = 4$ units, and area of $ABCD = 8$ sq. units, and $\angle BPC$ is acute, where P is point of intersection of diagonals AC and BD. Then

31. The length of other diagonal BD is

(a) $\frac{10}{3}$ (b) 2 (c) $\frac{20}{3}$ (d) $\frac{11\sqrt{2}}{3}$

32. The length of side AB is equal to

(a) $\frac{2\sqrt{58}}{3}$ (b) $\frac{4\sqrt{58}}{3}$ (c) $\frac{\sqrt{58}}{3}$ (d) $\sqrt{58}$

33. The length of BC is equal to

(a) $\frac{4\sqrt{10}}{3}$ (b) $\frac{2\sqrt{10}}{3}$ (c) $\frac{8\sqrt{10}}{3}$ (d) $\frac{\sqrt{10}}{3}$

Matrix-Match Type

34. Match the following :

Column-I		Column-II	
A.	The distance between the lines $(x+7y)^2+4\sqrt{2}(x+7y)-42=0$ is	p.	2
B.	If the sum of the distance of a point from two perpendicular lines in a plane is 1, then its locus is $\lvert x\rvert + \lvert y\rvert = k$, where k is equal to	q.	7
C.	If $6x + 6y + m = 0$ is acute angle bisector of line $x + 2y + 4 = 0$ and $4x + 2y - 1 = 0$, then m is equal to	r.	3
D.	Area of the triangle formed by the lines $y^2 - 9xy + 18x^2 = 0$ and $y = 6$ is	s.	1

35. Match the following :

Column-I		Column-II	
A.	The value of k for which $4x^2 + 8xy + ky^2 = 9$ is the equation of a pair of straight lines is	p.	3
B.	If the sum of the slopes of the lines given by $x^2 - 2cxy - 7y^2 = 0$ is four times their product, then the value of c is	q.	−3
C.	If the gradient of one of the lines $x^2 + hxy + 2y^2 = 0$ is twice that of the other, then $h =$	r.	2
		s.	4

SOLUTIONS

1. (c): Equation of one of the bisectors are $\frac{y-3x}{\sqrt{10}} = \pm\frac{y-x}{\sqrt{2}}$. We have to consider the bisector which is bisecting the internal angle of the triangle. For the internal bisector, the slope of the bisector should be positive.

$$\Rightarrow \left(\frac{1}{\sqrt{5}}+1\right)y - x\left(1+\frac{3}{\sqrt{5}}\right) = 0$$

Equation of second bisector is given by

$$\frac{y+3x-8}{\sqrt{10}} = \frac{y-3x}{\sqrt{10}} \Rightarrow x = \frac{4}{3}$$

$$\therefore \text{ Incentre } = \left(\frac{4}{3}, \left(\frac{3+\sqrt{5}}{1+\sqrt{5}}\right)\frac{4}{3}\right) = \left(\frac{4}{3}, \frac{2+2\sqrt{5}}{3}\right)$$

2. (d): We know that orthocentre is concurrency of altitudes and line joining orthocentre to vertex is perpendicular to the opposite side.

Let $A = (h, k)$, $m_{AH} = \frac{k-4}{h-1}$, $m_{OB} = \frac{4}{3}$,

where m_{AH} and m_{OB} are slopes of lines AH and OB respectively, which are perpendicular to each other.

$$\Rightarrow \frac{k-4}{h-1}\cdot\frac{4}{3} = -1 \Rightarrow 4k + 3h = 19 \qquad \text{...(i)}$$

Slope of OA *i.e.*, $m_{OA} = \frac{k}{h}$, $m_{BH} = \frac{4-4}{3-1} = 0$

Since OA and BH are mutually perpendicular $\Rightarrow h = 0$.

Putting $h = 0$ in (i), we get $k = \frac{19}{4}$

Thus coordinate of A is $\left(0, \frac{19}{4}\right)$.

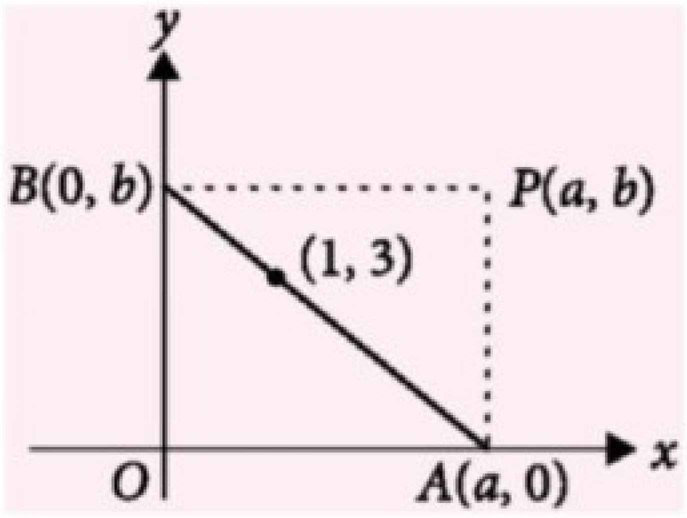
3. (a): Since the distance between the parallel lines $lx + my + n = 0$ and $lx + my + n' = 0$ is same as the distance between the parallel lines $mx + ly + n = 0$ and $mx + ly + n' = 0$. Therefore, the parallelogram is rhombus. Since the diagonals of a rhombus are at right angles, therefore the required angle is $\pi/2$.

4. (d): Let the lines represented by $ax^2 + by^2 + 2hxy = 0$ be $y = m_1x$ and $y = m_2x$, then $m_1 + m_2 = \frac{-2h}{b}$, $m_1m_2 = a/b$. If these lines are reflected about the x-axis, then their equation becomes $y + m_1x = 0$ and $y + m_2x = 0$ and their combined equation is $(y + m_1x)(y + m_2x) = 0$

$\Rightarrow y^2 + xy(m_1 + m_2) + m_1m_2x^2 = 0$

$\Rightarrow by^2 - 2hxy + ax^2 = 0$

5. (c) : Let the line be $\frac{x}{a}+\frac{y}{b}=1$. Since the line is passing through (1, 3),



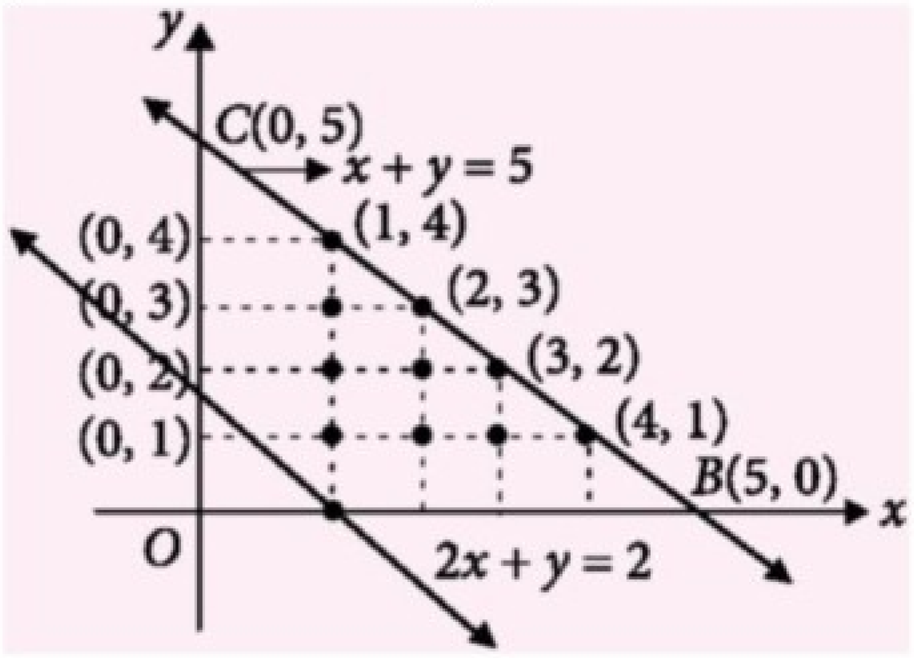
hence $\frac{1}{a}+\frac{3}{b}=1.$

Now, $A = (a, 0)$,
$B = (0, b)$ $\therefore$ $P = (a, b)$
Thus locus of 'P' is
$\frac{1}{x}+\frac{3}{y}=1$

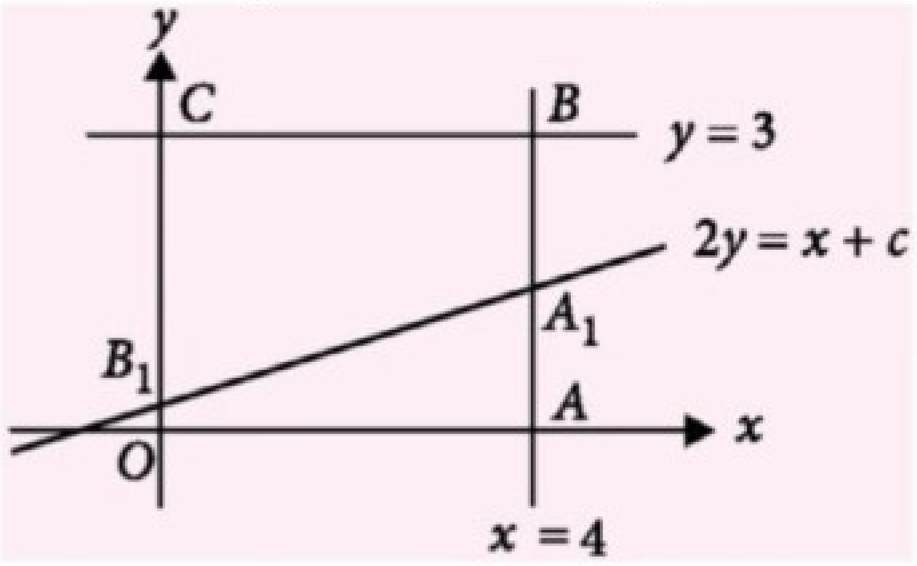
6. (b) : Since the given lines are concurrent. So, point of intersection of lines $x = \alpha + m$ and $y = -2$ which is $(\alpha + m, -2)$ will also satisfy $y = mx$
$\Rightarrow$ $m^2 + m\alpha + 2 = 0$
Since m is real $\therefore$ $\alpha^2 - 8 \geq 0$ $\Rightarrow$ $|\alpha|\geq 2\sqrt{2}.$
Hence the least value of $|\alpha|=2\sqrt{2}.$

7. (b) : Since the line L_1 and L_2 are perpendicular and angle between L_1 and L_3 is 45°.
Hence, the triangle will be right angled isosceles triangle.

8. (c) : Adjacent figure indicates that there are exactly six good points inside the quadrilateral $ABCD$.



9. (a) : Let the required line be $2y = x + c$.



We have $A_1\left(4, \frac{4+c}{2}\right), B_1\left(0, \frac{c}{2}\right)$
Area of rectangle $OABC$ is 12 sq. units
Thus area of trapezium OAA_1B_1 is
$6=\frac{1}{2}(4)\left(\frac{c}{2}+\frac{4+c}{2}\right)$ $\Rightarrow$ $c=1$
Thus required line is $2y = x + 1$

10. (b) : Equation of bisector is given by
$$\frac{x+y-1}{\sqrt{2}}=\pm\frac{7x-y-3}{\sqrt{50}}$$
Now at (2, 3), $x + y - 1 > 0$ and $7x - y - 3 > 0$.
Hence, the equation of bisector that contains the point (2, 3) is $\frac{x+y-1}{\sqrt{2}}=\frac{7x-y-3}{\sqrt{50}}$

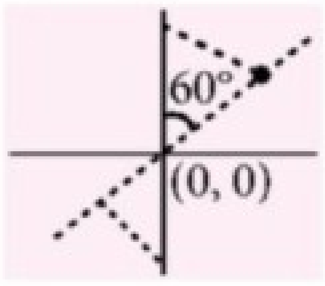
11. (c) : Let the point of intersection be (h, k)
Then $h\cos\alpha + k \sin\alpha = a$ ...(i)
and $h\sin\alpha - k\cos\alpha = b$ ...(ii)
Eliminating α and β from (i) and (ii), we get required locus as $x^2 + y^2 = a^2 + b^2$

12. (c) : Clearly $x^2 - 7xy - y^2 = 0$ will represent two perpendicular lines *i.e.*, triangle will be a right angled triangle and orthocentre will be the right angled vertex which is (0, 0).
Clearly distance of (0, 0) from $x + y - 2 = 0$ is $\sqrt{2}$.

13. (a) : Given lines are $y=\frac{\sqrt{3}+1}{\sqrt{3}-1}x$ and $y=\frac{\sqrt{3}-1}{\sqrt{3}+1}x$
Clearly inclination of both the lines with the positive direction of x-axis are 75° and 15°. Clearly the inclination of bisector will be 45°. Hence its equation is $y = x$.

14. (b) : Parametric equation of line $y=\sqrt{3}x$ is
$$\frac{x}{\frac{1}{2}}=\frac{y}{\frac{\sqrt{3}}{2}}=r$$
Any point on this line is $\left(\frac{r}{2}, \frac{\sqrt{3}r}{2}\right)$
At point of intersection, the point will also satisfy $x^3 + y^2 + 3x^2 + 8 = 0$
$\Rightarrow$ $\frac{r^3}{8}+\frac{3r^2}{4}+\frac{3r^2}{4}+8=0$ $\Rightarrow$ $r^3 + 12r^2 + 64 = 0$
$\Rightarrow OA\cdot OB\cdot OC = -64$

15. (a) : Clearly, the third vertex will lie on the y-axis. Hence the points are (0, 2) or (0, –2)



16. (c) : Let a and b are x and y intercept of line respectively. Then, area $=\frac{1}{2}ab=|\lambda|$ ...(i)
Equation of line passing through (2, 2)
$\frac{x}{a}+\frac{y}{b}=1\Rightarrow\frac{2}{a}+\frac{2}{b}=1$ ...(ii)
$\Rightarrow$ $2(a + b) = ab = 2|\lambda|$
$\Rightarrow$ $a + b = |\lambda|$ ...(iii) and $ab = 2|\lambda|$ ...(iv)
$\Rightarrow$ $a(|\lambda| - a) = 2|\lambda|$ [Using (iii) and (iv)]
$\Rightarrow$ $a^2 - a|\lambda| + 2|\lambda| = 0$
Intercepts on axes made by the line are given by $x^2 - |\lambda|\, x + 2|\lambda| = 0$

17. (a) : The equation $x^2 - y^2 + 2y = 1$ represents $x - y + 1 = 0$ and $x + y - 1 = 0$

Hence, equation of their angle bisectors are $x = 0$ and $y = 1$.

18. (c) : Inclination of QR is 60°. Clearly, angle bisector will be inclined at an angle of 120° with the x-axis.

19. (d) : Since the given triangle is equilateral. Hence incentre will coincides with centroid which is $\left(1, \frac{1}{\sqrt{3}}\right)$.

20. (a) : Since x_1, x_2, x_3 and y_1, y_2, y_3 are in G.P

$$\Rightarrow \frac{x_2}{x_1} = \frac{x_3}{x_2} = k_1 \text{ and } \frac{y_2}{y_1} = \frac{y_3}{y_2} = k_1$$

$$\text{Area} = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Hence the points are collinear.

21. (b) : The line PQ and PR will be inclined at an angle of 45° with the line $2x + y = 3$

$$\Rightarrow \quad 1 = \left|\frac{m+2}{1-2m}\right| \quad \Rightarrow \quad m = -\frac{1}{3}, 3$$

Hence the lines are $y - 1 = -\frac{1}{3}(x-2)$ and $y - 1 = 3(x - 2)$

Now, we get the equation of pair of lines by multiplying these two equations.

22. (d) : Let the point S be (h, k). According to given condition

$\{(h + 1)^2 + k^2\} + \{(h - 2)^2 + k^2\} = 2\{(h - 1)^2 + k^2\}$

$\Rightarrow$ $2h + 3 = 0$, which is a straight line parallel to y-axis.

23. (a, c) : Given, $x\cos\alpha + y\sin\alpha = P$

Since, axis is rotated through angle 'θ', then transformed equation is

$\cos\alpha(X\cos\theta - Y\sin\theta) + \sin\alpha(X\sin\theta + Y\cos\theta) = P$

$\Rightarrow X\cos(\alpha - \theta) + Y\sin\alpha(\alpha - \theta) = P \Rightarrow X\cos\beta + Y\sin\beta = P$

Hence, $\cos\beta = \cos(\alpha - \theta)$, $\sin\beta = \sin(\alpha - \theta)$

24. (b, d) : $\theta = \frac{1}{2}\tan^{-1}\left(\frac{2h}{a-b}\right) = \frac{1}{2}\tan^{-1}\left(\frac{4\sqrt{3}}{-4}\right)$

$$= \frac{1}{2}\tan^{-1}(-\sqrt{3})$$

$$\Rightarrow \quad \theta = \frac{1}{2}\left(\pi - \frac{\pi}{3}\right), \frac{1}{2}\left(-\frac{\pi}{3}\right) \Rightarrow \theta = \frac{\pi}{3}, -\frac{\pi}{6}$$

25. (a, c) : It is clear that diagonals of the rhombus will be parallel to the bisectors of the given lines and will pass through (1, 3). Equations of bisectors of the given lines are

$\frac{x+y-1}{\sqrt{2}} = \pm\left(\frac{7x-y-5}{5\sqrt{2}}\right)$ or $2x - 6y = 0$, $6x + 2y = 5$

Therefore, the equations of diagonals are $x - 3y + 8 = 0$ and $3x + y - 6 = 0$. Thus the required vertex will be the point where these lines meet the line $y = 2x$. Solving these lines we get possible coordinates as $\left(\frac{8}{5}, \frac{16}{5}\right)$ and $\left(\frac{6}{5}, \frac{12}{5}\right)$.

26. (a, b) : $6a^2 - 7a - 3 < 0 \Rightarrow a \in \left(-\frac{1}{3}, \frac{3}{2}\right)$

27. (a, b, c, d) : Let $x^2 + 2hxy + y^2 = 0$ be given by $y = m_1x$ and $y = m_2x$; $m_1 + m_2 = -2h$

Slope of $y + x = 0$ is -1

$$\tan\alpha = \left|\frac{m_1+1}{1-m_1}\right|, \quad \tan\alpha = \left|\frac{m_2+1}{1-m_2}\right|$$

$$\tan\alpha = \frac{m_1+1}{1-m_1} \text{ and } \tan\alpha = -\left(\frac{m_2+1}{1-m_2}\right)$$

(for +ve signs, in both gives the same value but $m_1 \neq m_2$)

$$\Rightarrow \quad m_1 = \frac{\tan\alpha - 1}{\tan\alpha + 1}, \quad m_2 = \frac{\tan\alpha + 1}{\tan\alpha - 1}$$

$$\Rightarrow \quad m_1 + m_2 = -2\sec 2\alpha \quad \Rightarrow \quad h = \sec 2\alpha$$

$$\Rightarrow \quad \cos 2\alpha = \frac{1}{h}, \quad \Rightarrow \quad 2\cos^2\alpha - 1 = \frac{1}{h}$$

$$\Rightarrow \quad \cos\alpha = \left(\frac{1+h}{2h}\right)^{\frac{1}{2}} \Rightarrow \quad \cot\alpha = \sqrt{\frac{h+1}{h-1}}$$

28. (a) : Statement-2 is true as the sum of the coefficients of x^2 and y^2 must be zero for lines to be perpendicular *i.e.*, $3c + 2m + 4 - c = 0 \Rightarrow c + m + 2 = 0$.

In statement-1, let the equation of the chord be $y = mx + c$, then equation of the pair of lines joining the origin to the points of intersection of the chord and the curve is

$$3x^2 - y^2 - 2x\left(\frac{y-mx}{c}\right) + 4y\left(\frac{y-mx}{c}\right) = 0$$

$\Rightarrow$ $(3c + 2m)x^2 - 2(1 + 2m)xy + (4 - c)y^2 = 0$

which are at right angles if $c + m + 2 = 0$ (using statement-2) and since the line $y = mx + c$ passes through $(1, -2)$, $c + m + 2 = 0$. So statement-1 is also true.

29. (a) : $\sum_{i=1}^{3} \frac{ax_i + by_i + c}{\sqrt{a^2+b^2}} = 0$

$$\Rightarrow \quad a\left(\sum x_i\right) + b\left(\sum y_i\right) + 3c = 0$$

$$\Rightarrow \quad a\frac{\sum_{i=1}^{3} x_i}{3} + b\frac{\sum_{i=1}^{3} y_i}{3} + c = 0 \Rightarrow \frac{5}{3}a + b + c = 0.$$

30. (a) : Since circumcentre is origin and $OA = OB = OC = r$

$OA = x_1^2 + x_1^2\tan^2\theta_1 = x_1\sec\theta_1$

$\therefore \quad x_1 = r\cos\theta_1$

Hence, $A(r\cos\theta_1, r\sin\theta_1)$, $B(r\cos\theta_2, r\sin\theta_2)$, $C(r\cos\theta_3, r\sin\theta_3)$.

Also, cicumcentre (O), centroid (G), and orthocentre (H) are collinear

$\Rightarrow$ slope of OH = slope of GO

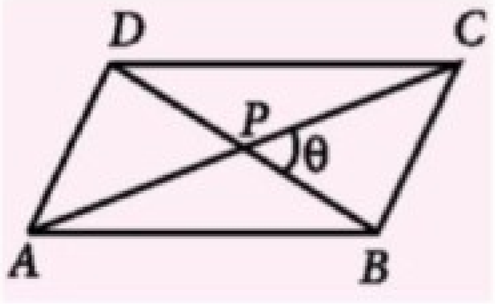
$$\Rightarrow \frac{y'-0}{x'-0}=\frac{(y\text{ coordinate of }G)-0}{(x\text{ coordinate of }G)-0}$$

$$\Rightarrow \frac{y'}{x'}=\frac{\sin\theta_1+\sin\theta_2+\sin\theta_3}{\cos\theta_1+\cos\theta_2+\cos\theta_3}$$

31. (c) **32. (a)** **33. (b)**

Let P be the point of intersection.

$$\tan\theta=\left|\frac{\frac{-1}{2}+2}{1+1}\right|=\frac{3}{4}\Rightarrow \sin\theta=3/5$$



$$\text{Area of }\Delta CPB=\frac{1}{2}PC\cdot PB\sin\theta=2\Rightarrow PB=\frac{10}{3}$$

$$\Rightarrow BD=\frac{20}{3}$$

$$\cos(\pi-\theta)=\frac{-4}{5}=\frac{4+\frac{100}{9}-AB^2}{2\times2\times\frac{10}{3}}\Rightarrow AB=\frac{2\sqrt{58}}{3}$$

$$\text{Again from }\Delta CPB,\ BC=\frac{2\sqrt{10}}{3}$$

34. A → p; B → s; C → q; D → r

(A) $(x+7y)^2+4\sqrt{2}(x+7y)-42=0$

$\Rightarrow (x+7y)[x+7y+7\sqrt{2}]-3\sqrt{2}(x+7y)-42=0$

$\Rightarrow (x+7y)[x+7y+7\sqrt{2}]-3\sqrt{2}(x+7y+7\sqrt{2})=0$

$\Rightarrow (x+7y+7\sqrt{2})(x+7y-3\sqrt{2})=0$

$\Rightarrow x+7y+7\sqrt{2}=0$ and $x+7y-3\sqrt{2}=0$

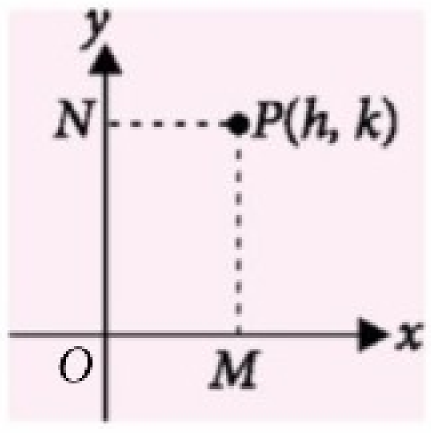
$$\Rightarrow d=\left|\frac{7\sqrt{2}+3\sqrt{2}}{\sqrt{1+49}}\right|=\frac{10\sqrt{2}}{\sqrt{50}}=2$$

(B) Let two perpendicular lines are coordinate axes.

Then, $PM + PN = 1 \Rightarrow h + k = 1$

Hence, the locus is $x + y = 1$.

But if the point lies in other quadrants, then $|x| + |y| = 1$. Hence, value of k is 1.



(C) Angle bisector between the lines $x + 2y + 4 = 0$ and $4x + 2y - 1 = 0$ is given by

$$\frac{x+2y+4}{\sqrt{1+4}}=\pm\frac{(-4x-2y+1)}{\sqrt{16+4}}$$

$$\Rightarrow x+2y+4=\pm\frac{(-4x-2y+1)}{2}$$

$\Rightarrow 2(x + 2y + 4) = \pm(-4x - 2y + 1)$

Since $aa' + bb' < 0$, so +ve sign gives acute angle bisector. Hence, $2x + 4y + 8 = -4x - 2y + 1$

$\Rightarrow 6x + 6y + 7 = 0 \Rightarrow m = 7$

(D) We have, $y^2 - 9xy + 18x^2 = 0$

or $y^2 - 6xy - 3xy + 18x^2 = 0$

$\Rightarrow y(y - 6x) - 3x(y - 6x) = 0$

$\Rightarrow y - 3x = 0$ and $y - 6x = 0$

The third line is $y = 6$. Therefore, area of the triangle formed by these lines,

$$=\frac{1}{2}\begin{vmatrix}0&0&1\\1&6&1\\2&6&1\end{vmatrix}=\frac{1}{2}|6-12|=3\text{ sq. units}$$

35. A → s; B → r; C → p, q

(A) The equation represents a pair of straight lines if $(4)(k)(-9) - (-9)(4)^2 = 0 \Rightarrow k = 4$

(B) Given, $m_1 + m_2 = 4m_1m_2$

$$\Rightarrow \frac{-2h}{b}=\frac{4a}{b}\Rightarrow -2(-c)=4\times1\Rightarrow c=2$$

(C) We have, $2m_2+m_2=\frac{-h}{2},\ 2m_2^2=\frac{1}{2}$

$$\Rightarrow 2\left(\frac{-h}{6}\right)^2=\frac{1}{2}\Rightarrow h^2=9\Rightarrow h=\pm3$$

◆◆

EXAM ALERT

JEE Advanced 2020 : IIT-Delhi to conduct exam on May 17, adds new centre in US

Indian Institute of Technology-Delhi will organise the Joint Entrance Examination (JEE-Advanced) in 2020 for admissions in 23 IITs all over the country. The examination will be conducted on Sunday, May 17. The IIT Joint Admission Board (JAB) has also decided to hold the JEE (Advanced) 2020 for the first time in the United States. The examination centre is being planned to set up in San Francisco.

Prof V Ramgopal Rao, Director, IIT Delhi, said, "JEE Advanced is an examination conducted by all IITs for admission to IITs. IITs have taken a conscious decision to encourage cultural diversity on their campuses. JEE exam is now conducted in five countries worldwide and we have added USA to this list. US has benefitted significantly by the presence of a large number of IIT alumni there and there is considerable awareness in the US about IITs. Conducting JEE in the bay area is a logical thing to do." The board has decided to shortlist 2.50 lakh students (including all categories) from JEE (Main) 2020 to appear for JEE (Advanced) 2020. On May 17, 2020, the first paper of JEE (Advanced) will start at 9.00 a.m. and end at 12 noon (IST). The second paper will be conducted from 2.30 p.m. to 5.30 p.m. (IST).

All those applicants who qualify JEE Advanced are eligible to get admission in the IITs, NITs.

Though Addis Ababa (Ethiopia), Colombo (Sri Lanka), Dhaka (Bangladesh), Dubai (UAE), Kathmandu (Nepal) and Singapore were among foreign centres where the JEE (Advanced) 2019 exam was held, it is for the first time will be conducted in the US. In 2019, IIT Roorkee was the organising institute for JEE (Advanced) in which a total number of 161,319 candidates had appeared in both papers 1 and 2. As many as 38,705 candidates were successful. Of the total qualified candidates, 5,356 were females.

CBSE warm-up!

CLASS-XI

Chapterwise Practice questions for CBSE Exams as per the latest pattern and marking scheme issued by CBSE for the academic session 2019-20.

Series 7

Conic Sections

Time Allowed : 3 hours
Maximum Marks : 80

GENERAL INSTRUCTIONS

(i) All the questions are compulsory.

(ii) This question paper contains 36 questions.

(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.

(iv) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted.

SECTION - A

Q.1 - Q.10 are multiple choice type questions. Select the correct option.

1. Find the equation of the circle which passes through the point (4, 5) and has its centre at (2, 2).

(a) $(x + 2)^2 + (y + 2)^2 = 13$

(b) $(x - 3)^2 + (y - 2)^2 = 13$

(c) $(x - 2)^2 + (y - 2)^2 = 13$

(d) $(x - 2)^2 + (y - 3)^2 = 13$

2. The equation of the circle having centre at (3, –4) and touching the line $5x + 12y - 12 = 0$ is

(a) $(x + 3)^2 + (y - 4)^2 = \left(\frac{45}{13}\right)^2$

(b) $(x + 2)^2 + (y + 4)^2 = \frac{45}{13}$

(c) $(x - 3)^2 + (y + 4)^2 = \left(\frac{45}{13}\right)^2$

(d) $(x - 3)^2 + (y + 4)^2 = \frac{15}{13}$

3. Find the equation of the parabola with focus (0, – 3) and directrix $y = 3$.

(a) $x^2 = -12y$ (b) $x^2 = 12y$

(c) $x^2 = 3y$ (d) $x^2 = -3y$

4. Find the coordinate of the focus of the parabola $y^2 = 12x$.

(a) (0, 3) (b) (–3, 0)

(c) (0, – 3) (d) (3, 0)

5. The eccentricity of the ellipse $12x^2 + 7y^2 = 84$ is equal to

(a) $\frac{\sqrt{5}}{7}$ (b) $\sqrt{\frac{5}{12}}$ (c) $\frac{\sqrt{5}}{12}$ (d) $\frac{5}{7}$

6. What is the eccentricity of hyperbola whose vertices and foci are (±2, 0) and (±3, 0) respectively?

(a) $\frac{4}{5}$ (b) $\frac{5}{4}$ (c) $\frac{3}{2}$ (d) $\frac{2}{3}$

7. The eccentricity of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ is

(a) $\frac{4}{5}$ (b) $\frac{5}{4}$ (c) $\frac{3}{2}$ (d) $\frac{2}{3}$

8. The centre of the circle $x^2 + y^2 + 2y = 0$ is

(a) (0, – 1) (b) (–1, 0)

(c) (0, 1) (d) (1, 0)

9. The equation of the parabola with vertex at (0, 0) and focus at (0, 2) is

(a) $y^2 = 8x$ (b) $x^2 = 8y$

(c) $x^2 = 2y$ (d) $y^2 = 2y$

10. The equation of the parabola whose focus is the point (2, 3) and directrix is the line $x - 4y + 3 = 0$ is

(a) $12x^2 - y^2 + 4xy - 78y = 0$

(b) $14x^2 + y^2 + 8xy - 4x + 2y - 4 = 0$

(c) $12x^2 + y^2 + 4xy - 78x = 0$

(d) $16x^2 + y^2 + 8xy - 74x - 78y + 221 = 0$

(Q.11 - Q.15) Fill in the blanks.

11. If the parabola $y^2 = 4ax$ passes through the point (3, 2), then the length of its latus rectum is ______ .

OR

The length of the latus rectum of the ellipse $9x^2 + 16y^2 = 144$ is ______ .

12. If the distance between the foci of a hyperbola is 16 and its eccentricity is 2, then equation of the hyperbola is ______ .

13. The area of the circle centred at (1, 2) and passing through (4, 6) is ______ .

14. The eccentricity of the curve $4x^2 + y^2 = 100$ is ______ .

15. The eccentricity of the hyperbola $-\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is ______ .

OR

The sum of the distances of any point on the ellipse $3x^2 + 4y^2 = 24$ from its foci is ______ .

(Q.16 - Q.20) Answer the following questions.

16. Write the equation of circle with centre $(-a, -b)$ and radius $\sqrt{a^2+b^2}$.

17. Find the equation of a parabola with vertex at origin and axis along x-axis and passing through the point (2, 8).

18. Show that line $x + 3y = 0$ is a diameter of the circle $x^2 + y^2 + 6x + 2y = 0$.

19. Find the equation of the ellipse with foci at (± 5, 0) and $x=\frac{36}{5}$ as one of the directices.

OR

Find the equation of the hyperbola with vertices (3, 0), (– 3, 0) and semi-latus rectum 4.

20. If the length of the major axis of an ellipse is 17/8 times the length of the minor axis, then find the eccentricity of the ellipse .

SECTION - B

21. Find the equation of a circle of radius 5 cm whose centre lies on x - axis and passes through the point (2, 3).

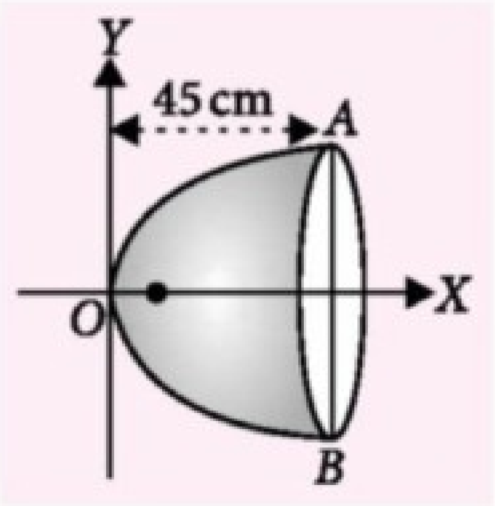
OR

Find the equation of the hyperbola whose eccentricity is $\sqrt{2}$ and the distance between the foci is 16, taking transverse and conjugate axes of the hyperbola as x and y-axes respectively.

22. If the eccentricity of an ellipse is $\frac{5}{8}$ and the distance between its foci is 10, then find the latus rectum of the ellipse.

23. Find the locus of the point of intersection of the lines $\sqrt{3}\,x - y - 4\sqrt{3}\,\lambda = 0$ and $\sqrt{3}\,\lambda x + \lambda y - 4\sqrt{3} = 0$ for different values of λ.

24. The focus of a parabolic mirror as shown in figure is at a distance of 5 cm from its vertex. If the mirror is 45 cm deep, then find the distance AB.



OR

Find the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, which passes through the points (3,0) and $(3\sqrt{2}, 2)$.

25. Find the equation of the ellipse, whose length of the major axis is 20 and foci are (0, ±5).

26. Find the equation of the hyperbola with foci (0, ±3) and vertices $\left(0, \pm\frac{\sqrt{11}}{2}\right)$.

SECTION - C

27. Find the equations of the circles touching y-axis at (0, 3) and making intercept of 8 units on the x-axis.

28. Prove that the equation of the parabola whose vertex and focus are on the x-axis at distances a' and a'' respectively from the origin is $y^2 = 4(a'' - a')(x - a')$.

OR

The foci of a hyperbola coincides with the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$. Find the equation of the hyperbola if its eccentricity is 2.

29. Find the equation of the ellipse that passes through the origin and has the foci at the points $(-1, 1)$ and $(1, 1)$.

30. Show that the equation of the circle which touches the coordinates axis and whose centre lies on the line $lx + my + n = 0$ is
$(l+m)(x^2+y^2)+2n\,(l+m)\,(x+y)+n^2=0$

31. Find the equation of the parabola whose latus-rectum is 4 units, axis is the line $3x + 4y - 4 = 0$ and the tangent at the vertex is the line $4x - 3y + 7 = 0$.

OR

Find the equation of the ellipse which passes through the point $(-3, 1)$ and has eccentricity $\frac{\sqrt{2}}{5}$, with x-axis as its major axis and centre at the origin.

32. If P is any point on a hyperbola and N is the foot of the perpendicular from P on the transverse axis, then prove that $\frac{(PN)^2}{(AN)(A'N)}=\frac{b^2}{a^2}$.

SECTION - D

33. Find the focus and the equation of the parabola whose vertex is $(6, -3)$ and directrix is $3x - 5y + 1 = 0$.

OR

Find the lengths and equations of major and minor axes, centre, eccentricity, foci, equation of directrices, vertices and length of latus rectums for the ellipse $16x^2 + y^2 = 16$.

34. Find the equation of the parabola with vertex $(2, -3)$ and focus $(0, 5)$.

35. A visitor with sign board 'DO NOT LITTER' is moving on a circular path in an exhibition. During the movement he stops at points represented by $(3, -2)$ and $(-2, 0)$. Also centre of the circular path is on the line $2x - y = 3$. What is the equation of the path?

OR

If e and e' be the eccentricities of a hyperbola and its conjugate, then prove that $\frac{1}{e^2}+\frac{1}{e'^2}=1$.

36. If S and S' be the foci, C the centre and P be any point on a rectangular hyperbola having lengths of transverse and conjugate axes equal, show that $SP{\cdot}S'P = CP^2$.

SOLUTIONS

1. **(c) :** As the circle is passing through the point (4, 5) and its centre is (2, 2) so its radius is $\sqrt{(4-2)^2+(5-2)^2}=\sqrt{13}$. Therefore the required equation of circle is $(x-2)^2+(y-2)^2=13$.

2. **(c) :** The perpendicular distance from centre $(3, -4)$ to the line $5x + 12y - 12 = 0$ is

$$d=\left|\frac{15-48-12}{\sqrt{25+144}}\right|=\frac{45}{13}\ \text{(radius)}$$

Hence, the required equation of the circle having centre at $(3, -4)$ is $(x-3)^2+(y+4)^2=\left(\frac{45}{13}\right)^2$

3. **(a) :** Since focus of the parabola is $(0, -3)$ and directrix $y = 3$.

$\therefore$ Equation of parabola will be $x^2 = -4\,ay$

i.e., $x^2 = -12y$. $[\because a = 3.]$

4. **(d) :** Given, parabola $y^2 = 12x = 4ax$

$\Rightarrow$ $4a = 12$ or $a = 3$ $\therefore$ Focus $= (3, 0)$

5. **(b) :** Given equation of ellipse is $12x^2 + 7y^2 = 84$

or $$\frac{x^2}{\left(\frac{84}{12}\right)}+\frac{y^2}{\left(\frac{84}{7}\right)}=1\Rightarrow\frac{x^2}{(\sqrt{7})^2}+\frac{y^2}{(\sqrt{12})^2}=1$$

Now, $$e=\sqrt{1-\frac{a^2}{b^2}}=\sqrt{1-\frac{7}{12}}=\sqrt{\frac{5}{12}}$$

6. **(c) :** Vertices are $(\pm 2, 0)$, foci are $(\pm 3, 0)$

$\Rightarrow$ $a = 2, c = 3$ $\therefore$ $e=\frac{c}{a}=\frac{3}{2}$.

7. **(a) :** Given, equation of ellipse is $\frac{x^2}{25}+\frac{y^2}{9}=1$

Major axis is along x-axis as $25 > 9$.

Here, $a^2 = 25, b^2 = 9$.

$\therefore$ Eccentricity (e) = $\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{9}{25}}=\frac{4}{5}$

8. **(a) :** Equation of circle is $x^2 + y^2 + 2y = 0$ ...(i)

General equation of circle is $x^2 + y^2 + 2gx + 2fy + c = 0$, with centre $(-g, -f)$

On comparing general equation with (i), we get $2g = 0, 2f = 2$ $\therefore g = 0, f = 1$

So, the centre of given circle is centre $(0, -1)$.

9. **(b):** Since the vertex is at (0, 0) and the focus is at (0, 2) which lies on y-axis, the y-axis is the axis of the parabola. Therefore, equation of the parabola is of the form $x^2 = 4ay$, thus, we have $x^2 = 4(2)y$, *i.e.*, $x^2 = 8y$.

10. **(d):** Using the definition of parabola, we have

$$\sqrt{(x-2)^2+(y-3)^2} = \left|\frac{x-4y+3}{\sqrt{17}}\right|$$

On squaring, we get

$$17(x^2 + y^2 - 4x - 6y + 13) = x^2 + 16y^2 + 9 - 8xy - 24y + 6x$$

or $\quad 16x^2 + y^2 + 8xy - 74x - 78y + 221 = 0$

11. **(b):** Since, the parabola $y^2 = 4ax$ passes through the point (3, 2).

$\therefore \quad (2)^2 = 4a(3) \Rightarrow 4 = 12a \Rightarrow a = \frac{1}{3}$

$\therefore \quad$ Length of latus rectum $= 4a = 4\times\frac{1}{3} = \frac{4}{3}$

OR

Given equation of ellipse is $9x^2 + 16y^2 = 144$

or $\quad \frac{9}{144}x^2 + \frac{16}{144}y^2 = 1 \quad$ or $\quad \frac{x^2}{(4)^2} + \frac{y^2}{(3)^2} = 1$

$\therefore \quad a = 4, b = 3.$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2\cdot 9}{4} = \frac{9}{2}$

12. Since distance between the foci is 16

$\Rightarrow \quad 2ae = 16 \Rightarrow ae = 8 \Rightarrow a = 4 \qquad [\because e = 2]$

Now, $b^2 = a^2(e^2 - 1) = (4)^2[(2)^2 - 1] = 16 \times 3 = 48$

Hence, the equation of hyperbola is $\frac{x^2}{16} - \frac{y^2}{48} = 1$

$\Rightarrow \quad 3x^2 - y^2 = 48$

13. Radius, $r = \sqrt{(4-1)^2+(6-2)^2} = \sqrt{9+16} = 5$

$\therefore \quad$ Required area of circle $= \pi r^2 = \pi(5)^2 = 25\pi$

14. Given, $4x^2 + y^2 = 100 \Rightarrow \frac{x^2}{25} + \frac{y^2}{100} = 1$

Here, $a^2 = 100, b^2 = 25$

$\therefore \quad c = \sqrt{a^2 - b^2} = \sqrt{100-25} = \sqrt{75} = 5\sqrt{3}$

$\therefore \quad e = \frac{c}{a} = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2}.$

15. We have, $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$

$\therefore a^2 = b^2(e^2 - 1) \Rightarrow e = \sqrt{\frac{a^2+b^2}{b^2}}.$

OR

Given equation of ellipse is $\frac{x^2}{8} + \frac{y^2}{6} = 1$

$\Rightarrow \quad a^2 = 8 \Rightarrow a = 2\sqrt{2}$

Sum of distances of any point on the ellipse from its foci $= 2a = 2\times 2\sqrt{2} = 4\sqrt{2}.$

16. The equation of circle with centre $(-a, -b)$ and radius $\sqrt{a^2+b^2}$ is given by

$$[x - (-a)]^2 + [y - (-b)]^2 = \left(\sqrt{a^2+b^2}\right)^2$$

$\Rightarrow \quad (x + a)^2 + (y + b)^2 = a^2 + b^2$

$\Rightarrow \quad x^2 + y^2 + 2ax + 2by = 0$

17. Let the equation of the parabola with vertex at origin and axis along x-axis is $y^2 = 4ax$ ...(i)

Since (i) passes through (2, 8).

$\therefore \quad (8)^2 = 4a(2) \Rightarrow a = 8$

Putting value of a in (i), we get required equation of parabola $y^2 = 32x$.

18. Given equation of the circle is

$x^2 + y^2 + 6x + 2y = 0$

Comparing with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get

Centre $= (-g, -f) = (-3, -1)$

Now, given equation of line is $x + 3y = 0$

$\Rightarrow \quad -3 - 3 = -6 \neq 0$

So, centre do not passes through the given line.

$\therefore \quad$ Given line is not the diameter of the circle.

19. We have $ae = 5$, $\frac{a}{e} = \frac{36}{5}$ which gives $a^2 = 36$ or $a = 6$. Therefore, $e = \frac{5}{6}$.

Now $b = a\sqrt{1-e^2} = 6\sqrt{1-\frac{25}{36}} = \sqrt{11}$. Thus, the equation of the ellipse is $\frac{x^2}{36} + \frac{y^2}{11} = 1.$

OR

We have $a = 3$ and $b^2/a = 4 \Rightarrow b^2 = 12$

Hence, the equation of the hyperbola be $\frac{x^2}{9} - \frac{y^2}{12} = 1$

$\Rightarrow \quad 4x^2 - 3y^2 = 36.$

20. Length of major axis $= \frac{17}{8} \times$ length of minor axis

$\Rightarrow \quad 2a = \frac{17}{8}\times 2b \Rightarrow \frac{b}{a} = \frac{8}{17}$

$\Rightarrow \quad e = \sqrt{1-\frac{64}{289}} = \sqrt{\frac{225}{289}} = \frac{15}{17}$

21. Let the coordinates of the centre of the required circle be $C\ (a, 0)$. Since it passes through $P\ (2, 3)$.

$\therefore \quad CP =$ radius $= 5$ cm

$\Rightarrow \quad \sqrt{(a-2)^2+(0-3)^2} = 5 \Rightarrow a - 2 = \pm 4$

$\Rightarrow \quad a = 6 \quad$ or $\quad a = -2$

Thus, the coordinates of the centre are (6, 0) or (–2, 0).

Hence, the equation of the required circle are

$(x - 6)^2 + (y - 0)^2 = 5^2$ and $(x + 2)^2 + (y - 0)^2 = 5^2$

$\Rightarrow \quad x^2 + y^2 - 12x + 11 = 0$ and $x^2 + y^2 + 4x - 21 = 0$

OR

Given, $2ae = 16 \Rightarrow 2 \cdot a\sqrt{2} = 16 \Rightarrow a = 4\sqrt{2}$

Also $b^2 = a^2(e^2 - 1) = 32(2 - 1) = 32$

Thus $a^2 = 32$ and $b^2 = 32$

$\therefore$ The required equation of hyperbola is $\frac{x^2}{32} - \frac{y^2}{32} = 1$ or $x^2 - y^2 = 32$

22. Let the equation of the required ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and let e be its eccentricity.

We have, $e = \frac{5}{8}$ and $2ae = 10 \Rightarrow ae = 5 \Rightarrow a = 8$

$\therefore \ b^2 = a^2(1 - e^2) \ \Rightarrow \ b^2 = 64\left(1 - \frac{25}{64}\right) = 39$

Hence, length of the latus rectum

$= \frac{2b^2}{a} = 2 \times \frac{39}{8} = \frac{39}{4}$

23. Let (h, k) be the point of intersection of the given lines. Then,

$\sqrt{3}h - k - 4\sqrt{3}\lambda = 0$ and $\sqrt{3}\lambda h + \lambda k - 4\sqrt{3} = 0$

$\Rightarrow \ \sqrt{3}h - k = 4\sqrt{3}\lambda$ and $\lambda(\sqrt{3}h + k) = 4\sqrt{3}$

$\Rightarrow \ (\sqrt{3}h - k)\lambda\ (\sqrt{3}h + k) = (4\sqrt{3}\lambda)(4\sqrt{3})$

$\Rightarrow \ 3h^2 - k^2 = 48$

Hence, the locus of (h, k) is $3x^2 - y^2 = 48$.

24. Since the distance from the focus to the vertex is 5 cm. We have, $a = 5$. If the origin is taken at the vertex and the axis of the mirror lies along the positive x-axis, the equation of the parabolic section is

$y^2 = 4(5)x = 20x$

We have, $x = 45$, so $y^2 = 900$

Therefore, $y = \pm 30$

Hence, $AB = 2y = 2 \times 30 = 60$ cm

OR

Given that the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is passing through the points $(3, 0)$ and $(3\sqrt{2}, 2)$, so we get $a^2 = 9$ and $b^2 = 4$.

Again, we know that $b^2 = a^2(e^2 - 1)$.

$\Rightarrow \ 4 = 9(e^2 - 1) \ \Rightarrow \ e^2 = \frac{13}{9} \ \Rightarrow \ e = \frac{\sqrt{13}}{3}$

25. Since the foci are on y-axis, the major axis is along the y-axis. So, equation of the ellipse is of the form $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$.

Given that a = semi-major axis $= \frac{20}{2} = 10$

and the relation $c^2 = a^2 - b^2$ gives $5^2 = 10^2 - b^2$ *i.e.*, $b^2 = 75$

Therefore, the equation of the ellipse is $\frac{x^2}{75} + \frac{y^2}{100} = 1$

26. Since the foci is on y-axis, the equation of the hyperbola is of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Since the vertices are $\left(0, \pm\frac{\sqrt{11}}{2}\right) \Rightarrow a = \frac{\sqrt{11}}{2}$

Also, foci are $(0, \pm 3)$; $c = 3$ and $b^2 = c^2 - a^2 = \frac{25}{4}$.

Therefore, the equation of the hyperbola is

$\frac{y^2}{\left(\frac{11}{4}\right)} - \frac{x^2}{\left(\frac{25}{4}\right)} = 1$, *i.e.*, $100y^2 - 44x^2 = 275$.

27. Let the equation of the circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)

Since circle (i) touches y-axis at $(0, 3)$

$\therefore \ 9 + 6f + c = 0$ and $f^2 = c$

$\therefore \ f^2 + 6f + 9 = 0 \ \Rightarrow \ (f + 3)^2 = 0 \ \Rightarrow \ f = -3$

$\therefore \ c = f^2 = 9$

Since, length of the intercept of circle (i) on x-axis is 8

$\therefore \ 2\sqrt{g^2 - c} = 8 \ \Rightarrow \ g^2 - c = 16$

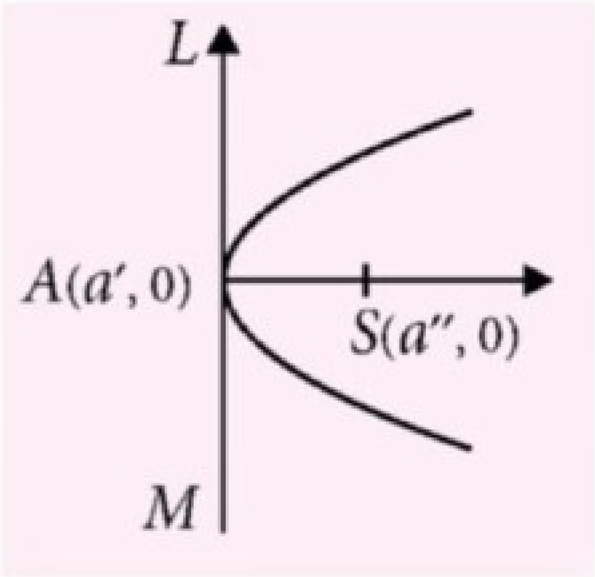
$\Rightarrow \ g^2 = c + 16 = f^2 + 16 = 9 + 16 = 25 \quad \therefore \ g = \pm 5$

Thus centre of the required circles are $(-g, -f)$ *i.e.*, $(-5, 3)$ and $(5, 3)$ and radius $= \sqrt{g^2 + f^2 - c} = \sqrt{25 + 9 - 9} = 5$

Hence, equation of required circles are

$(x + 5)^2 + (y - 3)^2 = 5^2$ and $(x - 5)^2 + (y - 3)^2 = 5^2$

i.e., $x^2 + y^2 + 10x - 6y + 9 = 0$ and $x^2 + y^2 - 10x - 6y + 9 = 0$



28. Since the vertex and focus of the parabola are on x-axis, therefore, axis of the parabola is x-axis. Let A be the vertex and S be the focus of the parabola. Since vertex and focus of the parabola are at distances a' and a'' from the origin, therefore $A \equiv (a', 0)$ and $S \equiv (a'', 0)$

We know that equation of a parabola whose vertex is (α, β) and axis is parallel to x-axis is $(y - \beta)^2 = 4a(x - \alpha)$, where a = distance between focus and vertex

Here $\alpha = a'$, $\beta = 0$ and $a = AS = a'' - a'$

$\therefore$ Required equation of the parabola will be

$y^2 = 4(a'' - a')(x - a')$.

OR

The given ellipse is $\frac{x^2}{25}+\frac{y^2}{9}=1$ or $\frac{x^2}{5^2}+\frac{y^2}{3^2}=1$

Here 5 > 3 ∴ $a = 5$ and $b = 3$

∴ Eccentricity of the ellipse,

$$e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{9}{25}}=\frac{4}{5}$$

∴ Foci of the ellipse are $(\pm ae, 0)$ or $(\pm 5\,(4/5), 0)$ or $(\pm 4, 0)$

Also centre of the hyperbola will be (0, 0).

Let the equation of the hyperbola be $\frac{x^2}{A^2}-\frac{y^2}{B^2}=1$

∴ Foci are $(\pm Ae, 0)$ or $(\pm 2A, 0)$ [$\because e = 2$]

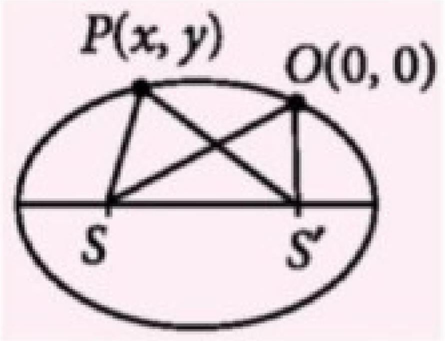
∴ $2A = 4 \Rightarrow A = 2$ and

$$B=A\sqrt{e^2-1}=2\sqrt{(2)^2-1}=2\sqrt{3}$$

∴ The equation of the hyperbola is

$$\frac{x^2}{(2)^2}-\frac{y^2}{(2\sqrt{3})^2}=1 \text{ or } \frac{x^2}{4}-\frac{y^2}{12}=1$$

29. Let $P(x, y)$ be any point on the ellipse and the foci be $S(-1, 1)$ and $S'(1, 1)$.



Now, P and O lies on the ellipse

$\Rightarrow PS + PS' = \text{constant} = OS + OS'$

[Since sum of focal distances of any point on the ellipse is constant]

$$\Rightarrow \sqrt{(x+1)^2+(y-1)^2}+\sqrt{(x-1)^2+(y-1)^2}=2\sqrt{2}$$

$$\Rightarrow \sqrt{(x+1)^2+(y-1)^2}=2\sqrt{2}-\sqrt{(x-1)^2+(y-1)^2}$$

$$\Rightarrow (x+1)^2+(y-1)^2=8+[(x-1)^2+(y-1)^2]-4\sqrt{2}\sqrt{(x-1)^2+(y-1)^2}$$

$$\Rightarrow 4x-8=-4\sqrt{2}\sqrt{(x-1)^2+(y-1)^2}$$

$$\Rightarrow x-2=-\sqrt{2}\sqrt{(x-1)^2+(y-1)^2}$$

$$\Rightarrow x^2-4x+4=2[x^2-2x+1+y^2-2y+1]$$

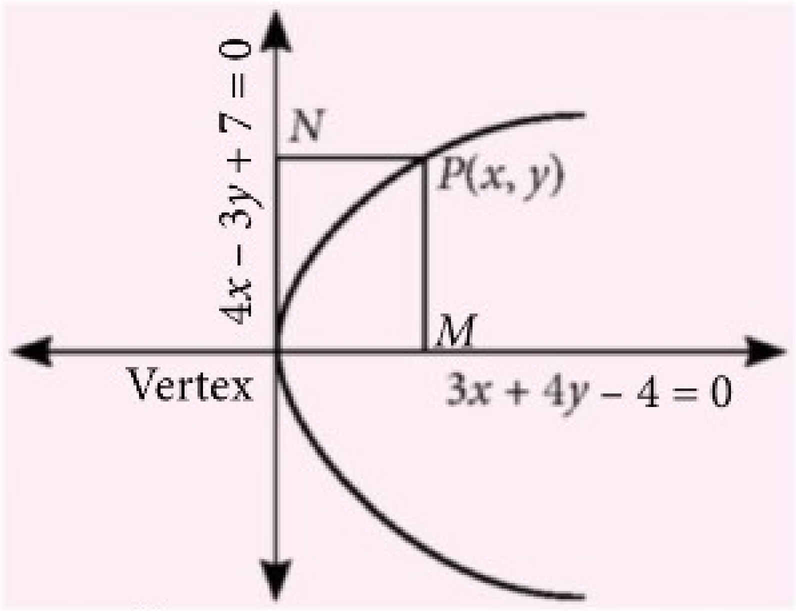
$\Rightarrow x^2+2y^2-4y=0$ is the required equation of the ellipse.

30. *Refer to answer 14, Page no. 245 of MTG CBSE Champion Mathematics, Class-11*

31. Let $P(x, y)$ be any point on the parabola and let PM and PN be perpendiculars from P on the axis and tangent at the vertex respectively. Then,

$PM^2 = (\text{Latus rectum})\,(PN)$

$$\Rightarrow \left(\frac{3x+4y-4}{\sqrt{3^2+4^2}}\right)^2=4\left(\frac{4x-3y+7}{\sqrt{4^2+(-3)^2}}\right)$$



$\Rightarrow (3x+4y-4)^2 = 20\,(4x-3y+7)$ is the equation of the required parabola.

OR

Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ be the equation of the ellipse passing through the point $(-3, 1)$.

Therefore, we have $\frac{9}{a^2}+\frac{1}{b^2}=1$.

or $9b^2+a^2=a^2b^2$

$$\Rightarrow 9a^2(1-e^2)+a^2=a^2a^2(1-e^2) \Rightarrow a^2=\frac{232}{23}$$

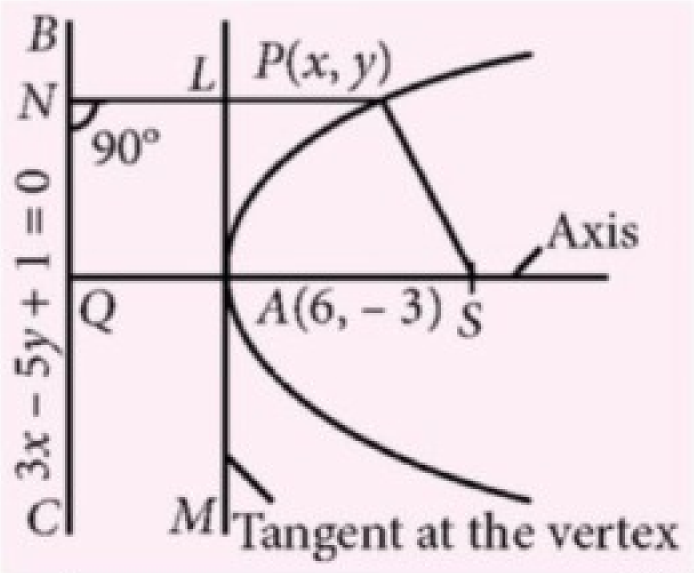
Again $b^2=a^2(1-e^2)=\frac{232}{23}\left(1-\frac{2}{25}\right)=\frac{232}{25}$

Hence, the required equation of the ellipse is

$$\frac{x^2}{\frac{232}{23}}+\frac{y^2}{\frac{232}{25}}=1 \Rightarrow 23x^2+25y^2=232.$$

32. *Refer to answer 117, Page no. 260 of MTG CBSE Champion Mathematics, Class-11*

33. Let $A(6, -3)$ be the vertex and BC be the directrix of the parabola. Let S be the focus.



Equation of directrix BC is

$3x - 5y + 1 = 0$ ...(i)

Equation of any line perpendicular to line (i) may be taken as $5x + 3y + k = 0$ ...(ii)

If line (ii) passes through $A(6, -3)$, then

$5 \times 6 + 3(-3) + k = 0$ or $k = -21$

∴ From (ii), equation of line AQ becomes

$5x + 3y - 21 = 0$ ...(iii)

Solving (i) and (iii) by cross multiplication method, we get $\frac{x}{105-3}=\frac{y}{5+63}=\frac{1}{9+25}$ ∴ $x = 3, y = 2$

∴ $Q \equiv (3, 2)$. Given $A \equiv (6, -3)$

Let $S \equiv (\alpha, \beta)$. Since A is the mid-point of QS

$$\therefore\ 6=\frac{3+\alpha}{2}\Rightarrow \alpha=9 \text{ and } -3=\frac{2+\beta}{2}\Rightarrow \beta=-8$$

∴ $S \equiv (9, -8)$

Let $P(x, y)$ be an arbitrary point on the parabola, then $PS = PN$

$$\Rightarrow\quad \sqrt{(x-9)^2+(y+8)^2}=\frac{|3x-5y+1|}{\sqrt{9+25}}$$

$$\Rightarrow\quad 25x^2+9y^2-618x+554y+30xy+4929=0$$

This is the required equation of the parabola.

OR

Given equation of the ellipse is

$$16x^2+y^2=16 \text{ or } \frac{x^2}{1^2}+\frac{y^2}{4^2}=1 \qquad \text{...(i)}$$

Here $4 > 1$ ∴ $a = 4$, $b = 1$

Length of major axis = $2a = 8$

Length of minor axis = $2b = 2$

Equation of major axis is $x = 0$

Equation of minor axis is $y = 0$

Coordinates of centre are $(0, 0)$

Eccentricity of the ellipse $e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{1}{16}}=\frac{\sqrt{15}}{4}$

Coordinates of foci are given by $y = \pm ae$, $x = 0$

or $y=\pm\sqrt{15}, x=0$

Hence foci are $(0,\pm\sqrt{15})$.

Equation of directrices are $y=\pm\frac{a}{e}$ or $y=\pm\frac{16}{\sqrt{15}}$

Coordinates of vertices are given by $(0, \pm 4)$.

Length of latus rectum = $\frac{2b^2}{a}=\frac{2\times 1}{4}=\frac{1}{2}$.

34. *Refer to answer 51, Page no. 251 of MTG CBSE Champion Mathematics, Class-11*

35. Let equation of circular path be

$(x - h)^2 + (y - k)^2 = r^2$ ...(i)

As circle passes through the points $(3, -2)$ and $(-2, 0)$.

∴ $(3 - h)^2 + (-2 - k)^2 = r^2$ ...(ii)

and $(-2 - h)^2 + (0 - k)^2 = r^2$ ...(iii)

Also, centre (h, k) lies on the line $2x - y = 3$.

∴ $2h - k = 3$ ...(iv)

From (ii) and (iii), we have

$$(3-h)^2+(-2-k)^2=(-2-h)^2+k^2$$

$\Rightarrow -10h+4k+9=0 \Rightarrow 10h-4k=9$ ...(v)

Solving (iv) and (v), we get, $h=-\frac{3}{2}, k=-6$

Substituting for h, k in (iii), we get

$$r^2=\left(-2+\frac{3}{2}\right)^2+(6)^2=\frac{145}{4}\Rightarrow r=\sqrt{\frac{145}{4}}$$

Substituting the values of h, k and r in (i), we get equation of circle as

$$\left(x+\frac{3}{2}\right)^2+(y+6)^2=\frac{145}{4} \Rightarrow x^2+y^2+3x+12y+2=0$$

OR

Refer to answer 125, Page no. 262 of MTG CBSE Champion Mathematics, Class-11

36. Let the equation of the rectangular hyperbola be

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad \text{...(i)}$$

Given, $a = b$, therefore (i) becomes $x^2 - y^2 = a^2$ ...(ii)

Now, $e=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{1+\frac{a^2}{a^2}}=\sqrt{2}$

Thus $e=\sqrt{2}$, $C\equiv(0, 0)$, $S\equiv(\sqrt{2}\,a, 0)$, $S'\equiv(-\sqrt{2}\,a, 0)$

Let $P \equiv (\alpha, \beta)$. Since P lies on (ii)

∴ $\alpha^2 - \beta^2 = a^2$ ...(iii)

Now, $SP^2 = (\sqrt{2}\,a-\alpha)^2+\beta^2=2a^2+\alpha^2+\beta^2-2\sqrt{2}\,a\,\alpha$

and $S'P^2 = (-\sqrt{2}\,a-\alpha)^2+\beta^2=2a^2+\alpha^2+\beta^2+2\sqrt{2}\,a\,\alpha$

Now, $SP^2 \cdot S'P^2 = (2a^2+\alpha^2+\beta^2)^2-8a^2\alpha^2$

$= 4a^4+4a^2(\alpha^2+\beta^2)+(\alpha^2+\beta^2)^2-8a^2\alpha^2$

$= 4a^2(a^2-2\alpha^2)+4a^2(\alpha^2+\beta^2)+(\alpha^2+\beta^2)^2$

$= 4a^2(\alpha^2-\beta^2-2\alpha^2)+4a^2(\alpha^2+\beta^2)+(\alpha^2+\beta^2)^2$ [from (iii)]

$= (\alpha^2+\beta^2)^2=(CP^2)^2=CP^4$ ∴ $SP\cdot S'P=CP^2$

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Ramanujan prize for U.K. mathematician

Harper's brilliant work recognised

The SASTRA Ramanujan prize for 2019 will be awarded to mathematician Adam Harper, Assistant Professor with the University of Warwick, England. The prize carries a citation and an award of $10,000 and is conferred annually on mathematicians from across the world who are less than 32 years of age, working in an area influenced by the genius Srinivasa Ramanujan.

"The SASTRA-Ramanujan Award has gained global repute ever since it was instituted in 2005 and today is easily amongst the top five awards of this type for mathematics. Many previous winners, including Manjul Bhargava and Akshay Venkatesh, went on to win the Fields Medal later," said Dr. S. Vaidhyasubramaniam, Vice-Chancellor of SASTRA University, in a press release.

Four mathematicians who were awarded this prize have gone on to win the Fields Medal later.

Noted number theorist and former director of the Institute of Mathematical Sciences, Chennai, Prof. R. Balasubramanian observed that the problems on which Mr. Harper works are interesting. "He has improved earlier results obtained by Kannan Soundararajan and Andrew Granville. His paper with Ben Green on large sieve is also worth mentioning," he said.

The citation said Mr. Harper was awarded the prize "for several outstanding contributions to analytic and probabilistic number theory."

"The prize recognizes his marvellous 2012 PhD thesis at Cambridge University, and his paper of 2013 in Crelle's Journal ...Harper's recent brilliant proof of a conjecture of Helson...and a related paper... to appear in Algebra and Number Theory. The prize also recognizes Harper's seminal work using the Riemann Hypothesis to determine the correct order upper bound for the higher moments of the Riemann zeta function on the critical line..."

The age limit is 32 years to commemorate the fact that Srinivasa Ramanujan accomplished a phenomenal body of work in this short span. Every year, this prize is awarded by SASTRA University on its campus near Kumbakonam in Tamil Nadu, on Ramanujan's birth anniversary, December 22.

MONTHLY TEST DRIVE

XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 6 : Sequence and Series

Time Taken : 60 Min.

Only One Option Correct Type

1. If 1, $\log_y x$, $\log_z y$, $-15 \log_x z$ are in A.P., then which of the following relations is incorrect?

(a) $x = z^3$ (b) $x = y^{-1}$ (c) $y = z^{-3}$ (d) $y = z^3$

2. If $x > 1, y > 1, z > 1$ are in G.P., then

$$\frac{1}{1+\ln x}, \frac{1}{1+\ln y}, \frac{1}{1+\ln z} \text{ are in}$$

(a) A.P. (b) H.P.

(c) G.P. (d) None of these

3. Let $S \subset (0, \pi)$ denote the set of values of x satisfying the equation

$$8^{1+|\cos x|+\cos^2 x+|\cos^3 x|+\ldots \text{ to } \infty} = 4^3$$

Then, $S =$

(a) $\left\{\frac{\pi}{3}\right\}$ (b) $\left\{\frac{\pi}{3}, -\frac{2\pi}{3}\right\}$

(c) $\left\{-\frac{\pi}{3}, \frac{2\pi}{3}\right\}$ (d) $\left\{\frac{\pi}{3}, \frac{2\pi}{3}\right\}$

4. If A.M., G.M. and H.M. of the first and last terms of the series 100, 101, 102, ..., $n - 1$, n are the terms of the series itself, then the value of n is ($100 < n \leq 500$)

(a) 200 (b) 300 (c) 400 (d) 500

5. Let a_1, a_2, a_3, ... be terms of an A.P.

If $\frac{a_1+a_2+\ldots+a_p}{a_1+a_2+\ldots+a_q} = \frac{p^2}{q^2}$, $p \neq q$, then $\frac{a_6}{a_{21}}$ equals

(a) $\frac{41}{11}$ (b) $\frac{7}{2}$ (c) $\frac{2}{7}$ (d) $\frac{11}{41}$

6. The sum of the infinite terms of the series

$$\frac{5}{3^2 \cdot 7^2} + \frac{9}{7^2 \cdot 11^2} + \frac{13}{11^2 \cdot 15^2} + \ldots \text{ is}$$

(a) $\frac{1}{18}$ (b) $\frac{1}{36}$ (c) $\frac{1}{54}$ (d) $\frac{1}{72}$

One or More Than One Option(s) Correct Type

7. If p, q and r are in A.P., then which of the following is/are true?

(a) p^{th}, q^{th} and r^{th} terms of A.P. are in A.P.

(b) p^{th}, q^{th} and r^{th} terms of G.P. are in G.P.

(c) p^{th}, q^{th} and r^{th} terms of H.P. are in H.P.

(d) None of these

8. If a, b, c are real and for some real x, $(a^2 + b^2)x^2 + 2(ab + bc)x + (b^2 + c^2) \leq 0$, then

(a) a, b, c are in G.P. (b) a, b, c are in A.P.

(c) a, b, c are in H.P. (d) None of these

9. Let S_1, S_2, ... be squares such that for each $n \geq 1$, the length of a side of S_n equals the length of a diagonal of S_{n+1}. If the length of a side S_1 is 10 cm, then for which of the following values of n is the area of S_n less than 1 sq. cm ?

(a) 7 (b) 8 (c) 9 (d) 10

10. The sum of n terms of the series

$$\frac{1}{1\cdot2\cdot3\cdot4} + \frac{1}{2\cdot3\cdot4\cdot5} + \frac{1}{3\cdot4\cdot5\cdot6} + \ldots. \text{ is}$$

(a) $\frac{n(n^2+6n+11)}{18(n+1)(n+2)(n+3)}$

(b) $\frac{n^3+6}{18(n+1)(n+2)(n+3)}$

(c) $\frac{1}{18}-\frac{1}{3(n+1)(n+2)(n+3)}$

(d) $\frac{1}{6}-\frac{1}{2(n+1)(n+2)(n+3)}$

11. If A_1, A_2; G_1, G_2 and H_1, H_2 are two arithmetic, geometric and harmonic means, respectively, between two quantities a and b, then ab is equal to

(a) A_1H_2

(b) A_2H_1

(c) G_1G_2

(d) None of these

12. If $a > 0$, $b > 0$, $c > 0$ and $2a + b + 3c = 1$ then

(a) $a^4b^2c^2$ is greatest when $a=\frac{1}{4}$

(b) $a^4b^2c^2$ is greatest when $b=\frac{1}{4}$

(c) $a^4b^2c^2$ is greatest when $c=\frac{1}{12}$

(d) greatest value of $a^4b^2c^2$ is $\frac{1}{9\cdot 4^8}$

13. Let a_1, a_2, a_3, a_4, ... be in G.P. If the H.M. of a_1 and a_2 is 12 and that of a_2 and a_3 is 36, then

(a) $a_1 = 8$ (b) $a_2 = 24$

(c) $a_3 = 72$ (d) $a_4 = 216$

Comprehension Type

Let A_1, A_2, A_3, ..., A_m be the arithmetic means between -2 and 1027 and G_1, G_2, G_3, ..., G_n be the geometric means between 1 and 1024. The product of geometric means is 2^{45} and the sum of arithmetic means is 1025×171.

14. The value of $\sum_{r=1}^{n} G_r$ is

(a) 512 (b) 2046

(c) 1022 (d) None of these

15. The numbers $2A_{171}$, $G_5^2 + 1$, $2A_{172}$ are in

(a) A.P. (b) G.P.

(c) H.P.

(d) None of these

Matrix Match Type

16. Match the following columns:

	Column-I	Column-II	
P.	If a, b, c are non-zero real numbers such that $3(a^2 + b^2 + c^2 + 1)$ $= 2 \times (a + b + c + ab + bc + ca)$, then a, b, c are in	1.	A.P.
Q.	If the square of differences of three numbers be in A.P., then their differences are in	2.	G.P.
R.	If $a - b$, $ax - by$, $ax^2 - by^2 (a, b \neq 0)$ are in G.P., then x, y, $\frac{ax-by}{a-b}$ are in	3.	H.P.

	P	Q	R
(a)	2	1	3
(b)	3	2	1
(c)	2,3	1	2
(d)	1,2	3	1,2

Numerical Value Type

17. If x_1, x_2, x_3, ..., x_{2008} are in H.P. and $\sum_{i=1}^{2007} x_i x_{i+1} = \lambda\, x_1 x_{2008}$, then λ must be equal to

18. After striking a floor a certain ball rebounds $\left(\frac{4}{5}\right)^{\text{th}}$ of the height from which it has fallen. The total distance that it travels before coming to rest if it is gently dropped from a height of 120 m is

19. The number of terms common between the series $1 + 3 + 9 + 27 + ...$ to 11 terms and $1 + 5 + 9 + 13 + ...$ to 1000 terms is

20. Let a, b, c be positive integers such that $\frac{b}{a}$ is an integer. If a, b, c are in geometric progression and the arithmetic mean of a, b, c is $b + 2$, then the value of $\frac{a^2+a-14}{a+1}$ is

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No. of questions correct ......
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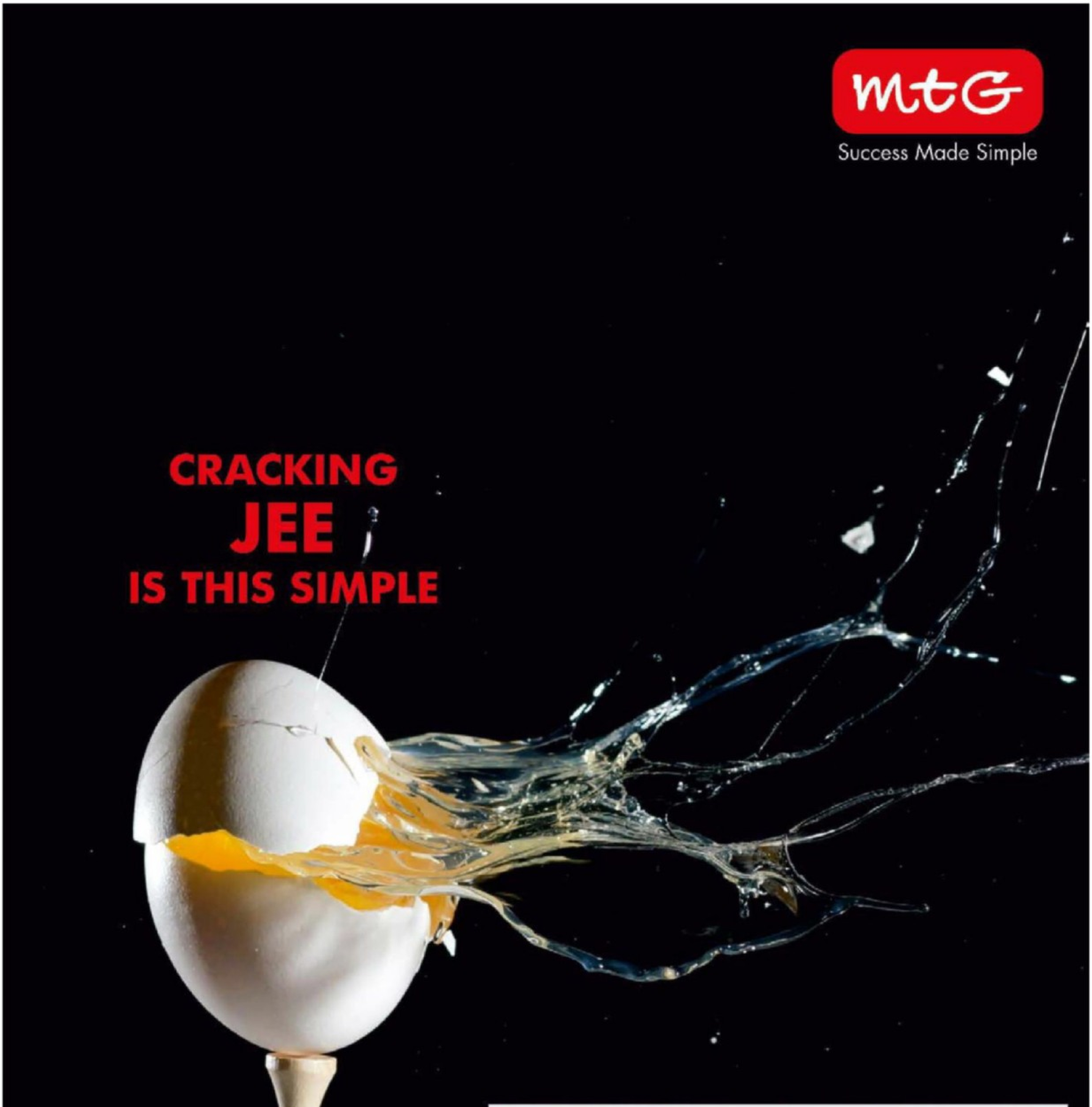
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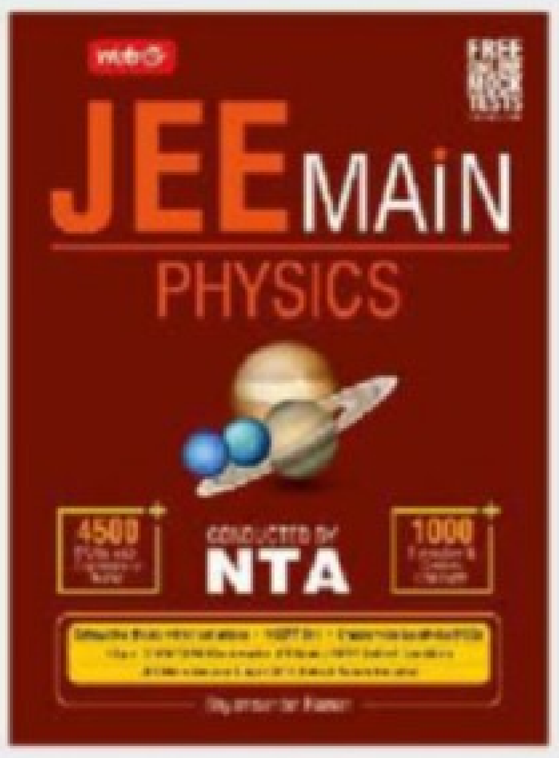
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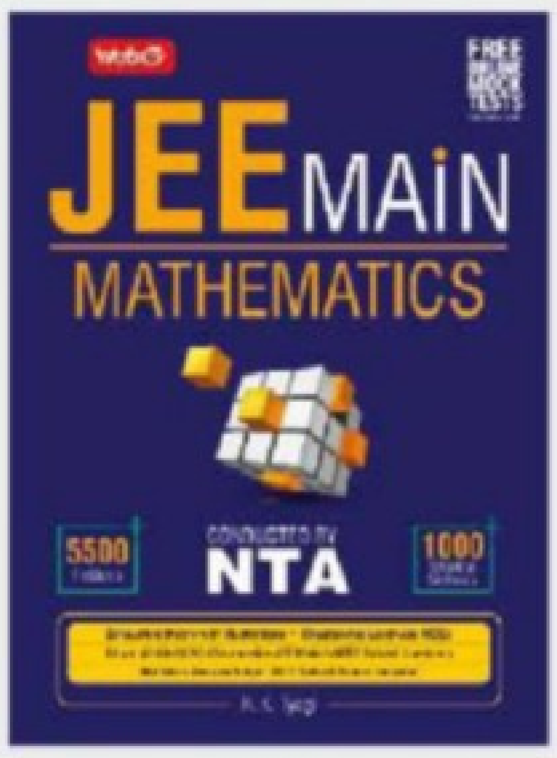
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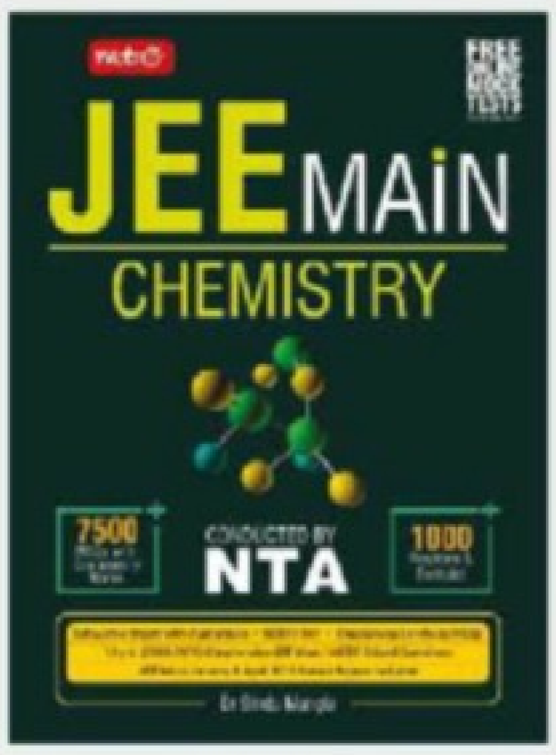
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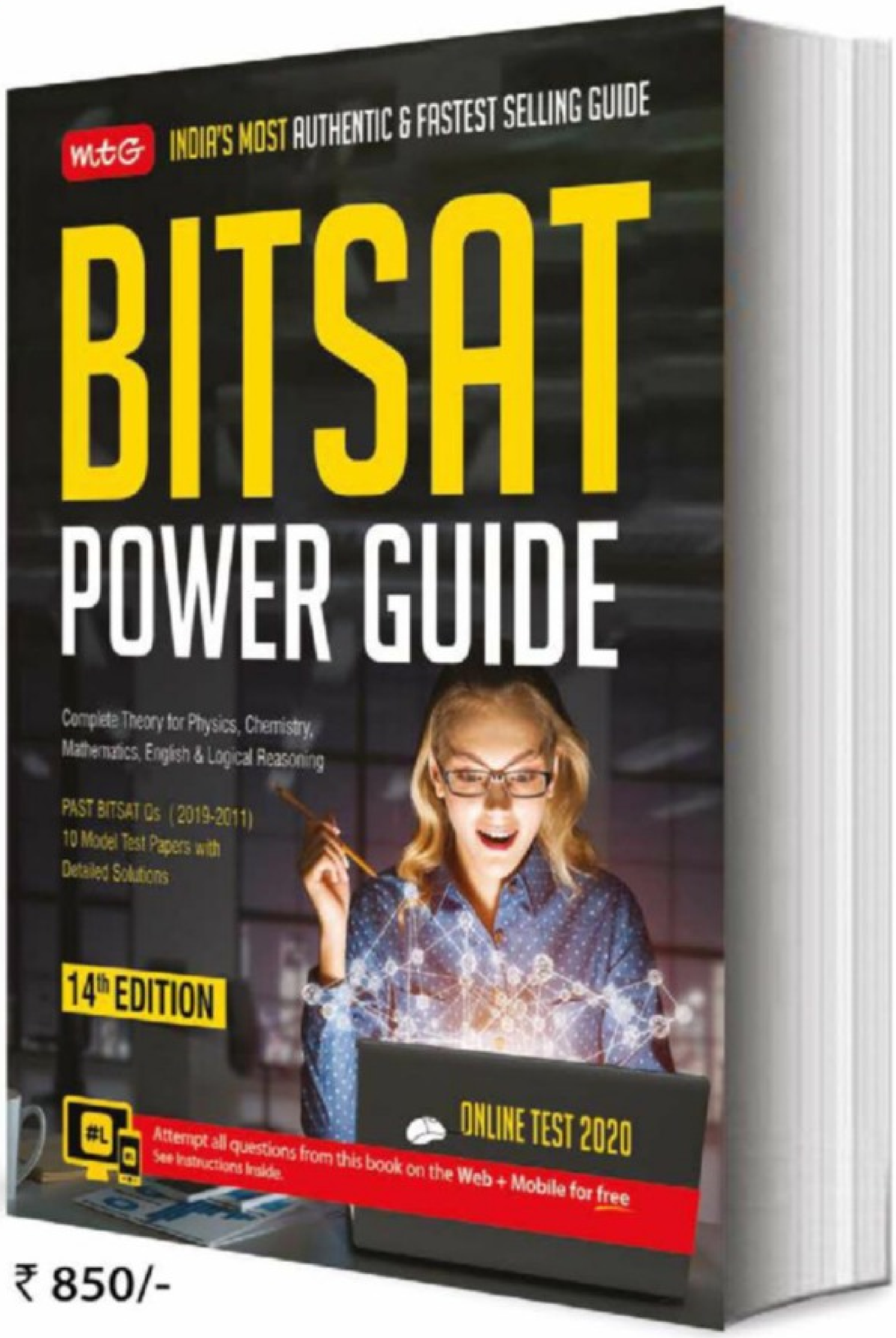
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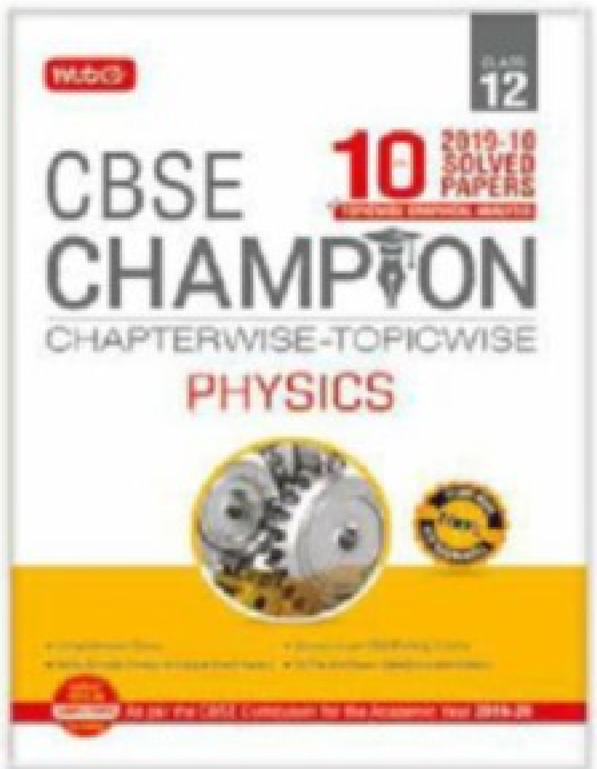
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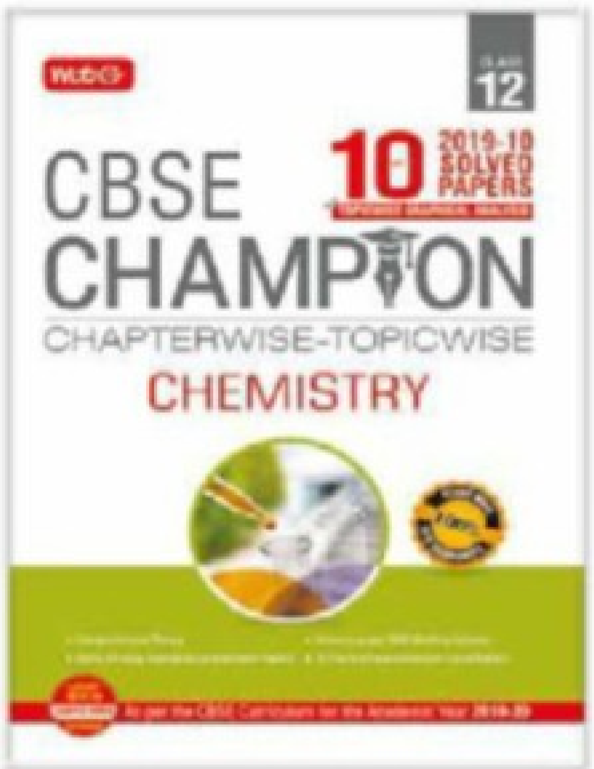
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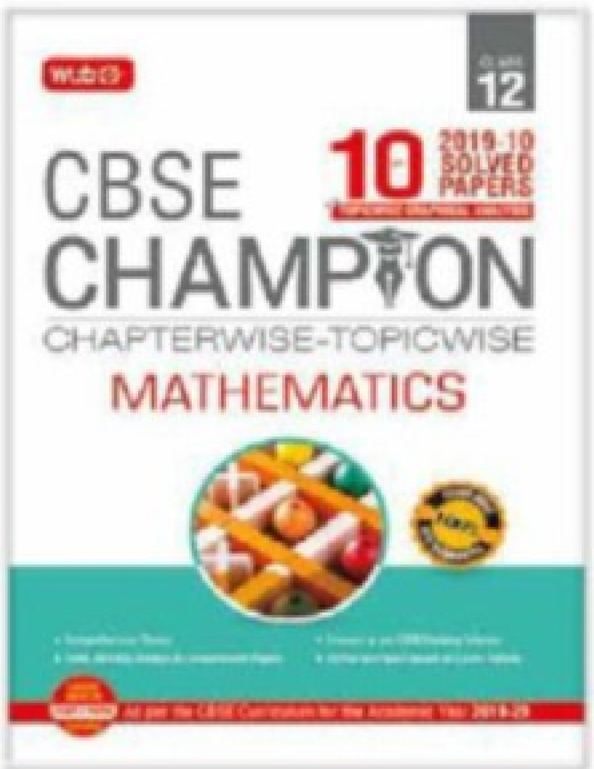
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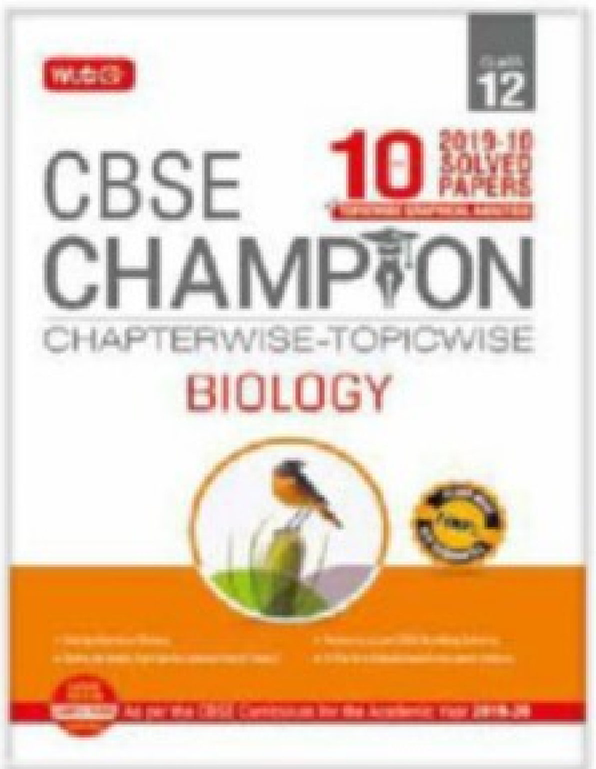
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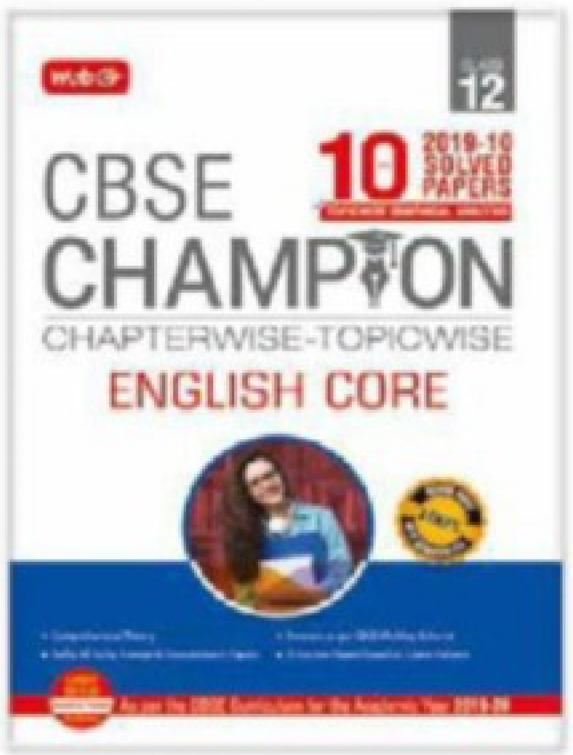
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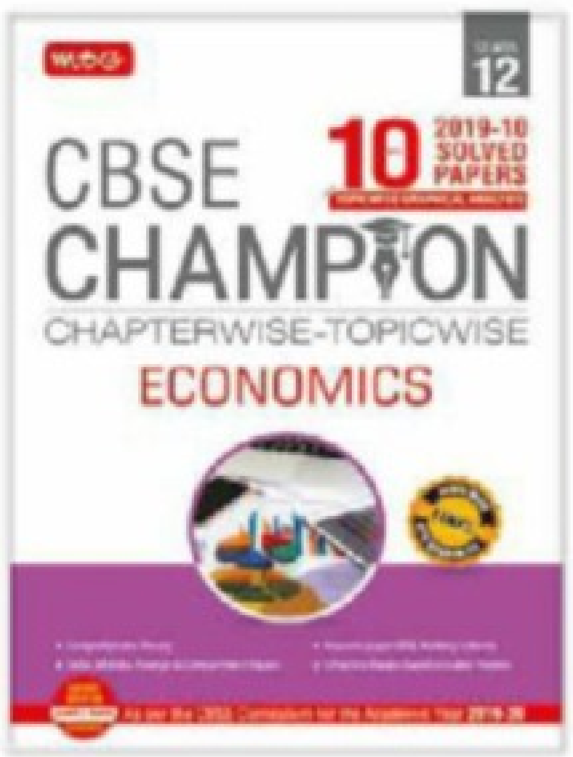
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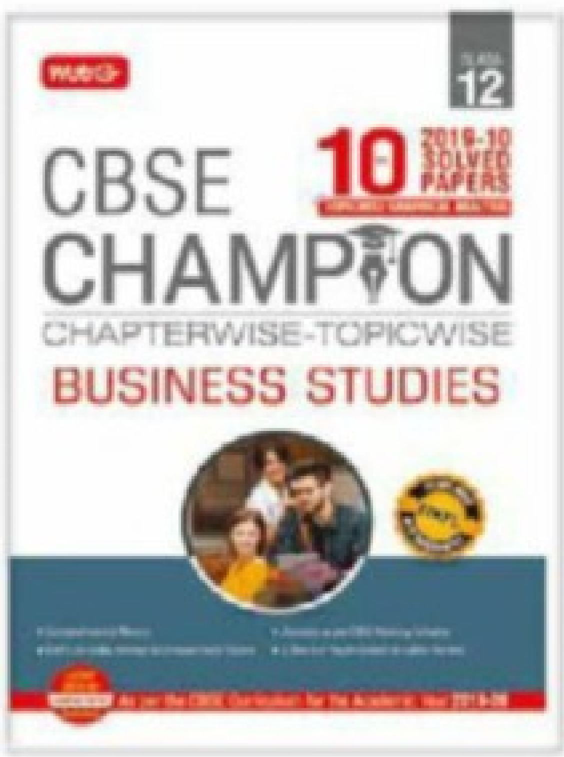
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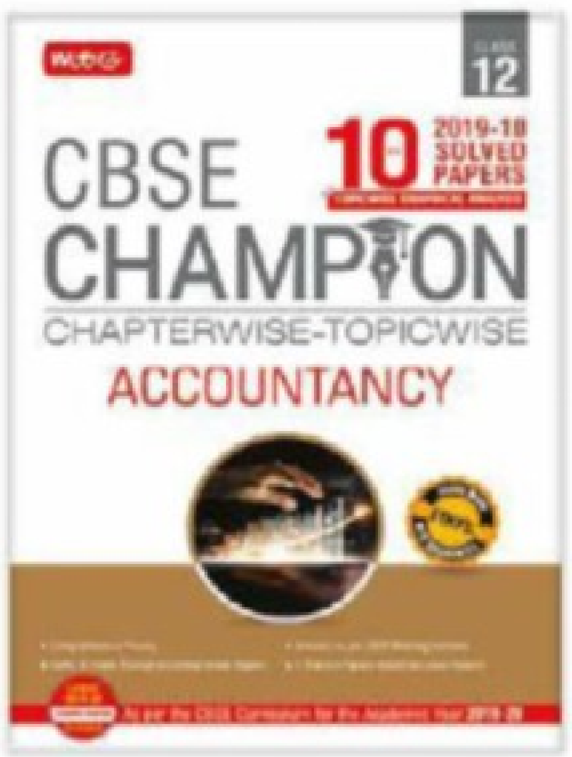
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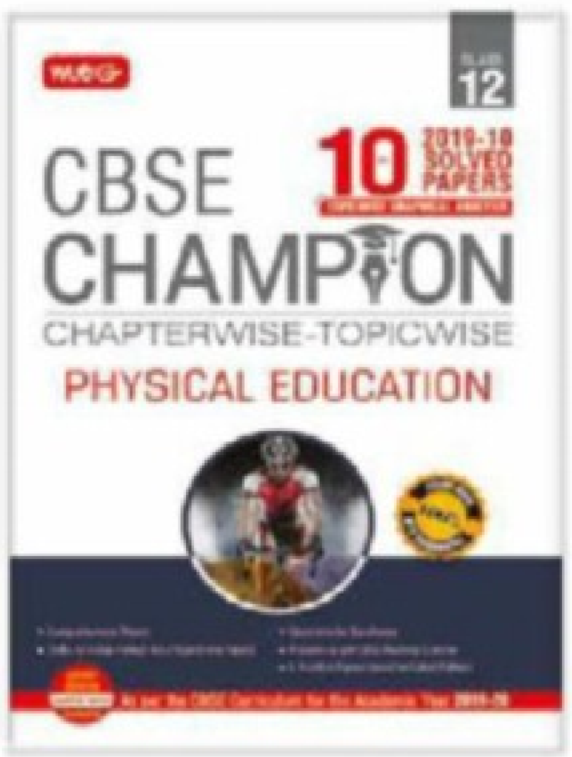
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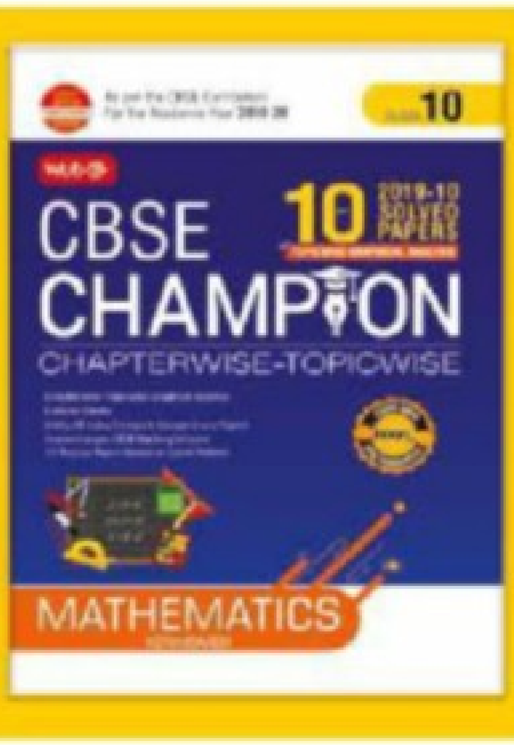
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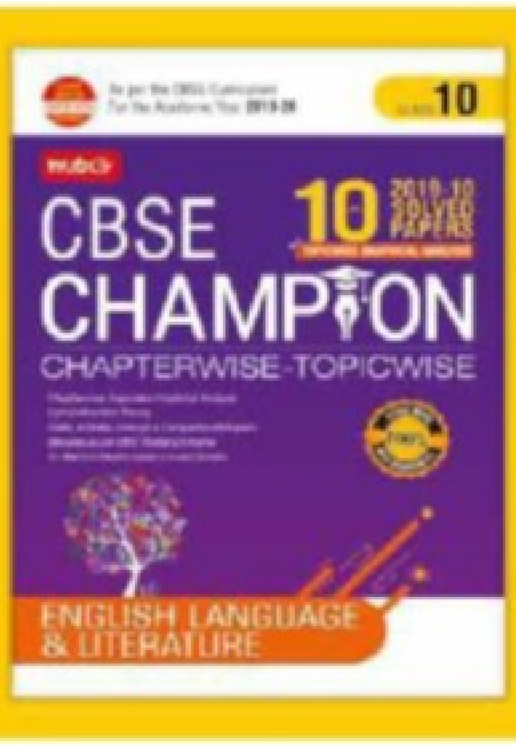
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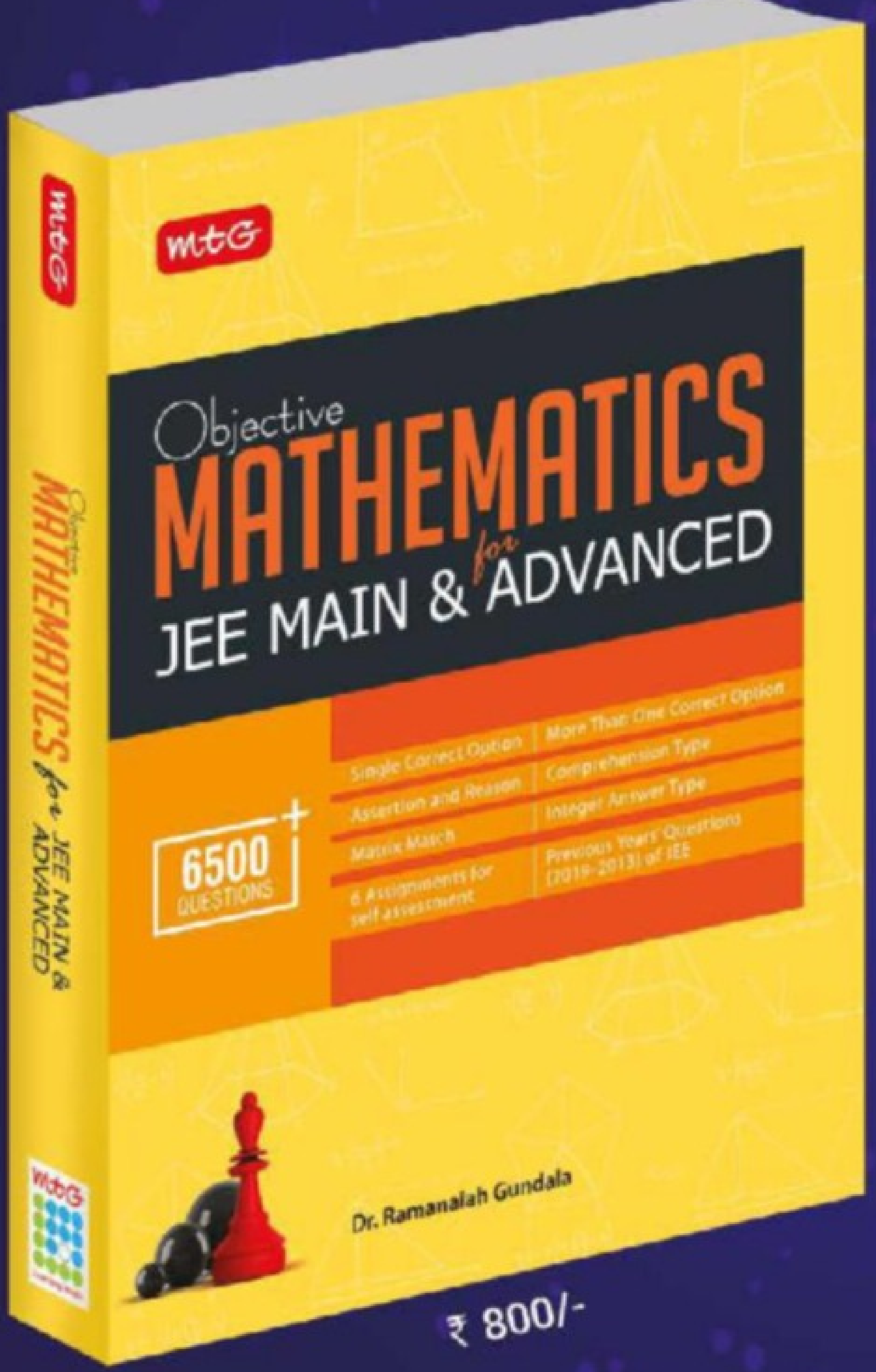
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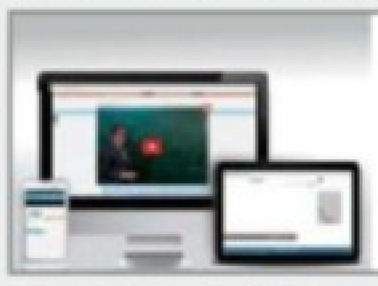

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